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SHEAR RESISTANCE OF STEEL FIBER REINFORCED CONCRETE BEAMS

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Abstract

Shear failure of structural members can be sudden and catastrophic. This is a typical failure at critical sections where, due to construction constraints, little or no shear reinforcing steel can be placed. The use of fibers in concrete matrix can partly solve the problem of an adequate reinforcement in such cases, because, in general, fibers enhance the tensile strength and ductility of concrete. In the paper, some analytical and empirical models for predicting the average shear cracking force and the average ultimate shear resistance of steel fiber reinforced concrete beams, as well as their evaluation with experimental results and of numerical simulation based on FEM are presented. Results obtained from the numerical simulation give good agreement with experimental results, so the presented computational models could be used for a proper simulation of the behavior of concrete beams subject to shear force and predicting their shear resistance or shear cracking force.

Keywords: steel fiber reinforced concrete; cracking force; shear resistance; FEM.

1 Introduction

Similar to vertical stirrups, fibers, in general, increase the shear resistance of beams, ductile behavior of the concrete matrix, and lead to an increase in their energy absorption capacity. Fibers have two important advantages over stirrups. Firstly, the fiber spacing is much smaller than the practical spacing of stirrups. This is beneficial because it helps fibers to bridge cracks in the whole of concrete volume. Secondly, the use of an adequate amount of fibers increases tensile strength of the concrete matrix, which increases cracking shear force. This fact leads not only to increasing of the beam shear resistance, but it can be also very important in case of shear connection by means perfobond strips.

2 Experiment

The results of the investigation in [1] were used to evaluate the accuracy of design formulas for shear strength of fiber reinforced concrete beams. In [1], a total of *16* beams were tested (Fig. 1) which were divided into four series designated as *A*, *B*, *C* a *D* by the percentages *0, 0.5, 0.75* and *1.0 %* of fiber volume in the concrete matrix (Tab. 1).

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In each of these series, the shear span-effective depth of beam ratio *a/d* varied from *2.0* to *4.4* at increments of *0.8*. The steel had the yield strength of *463 MPa*. The length and diameter of fibers used in the investigation were *30 mm* and *0.5 mm*, respectively. The ultimate tensile strength of fibers was *1260 MPa.* The concrete mix was made with ordinary Portland cement, natural sand, and crushed granite of *20 mm* maximum size (Tab. 1).

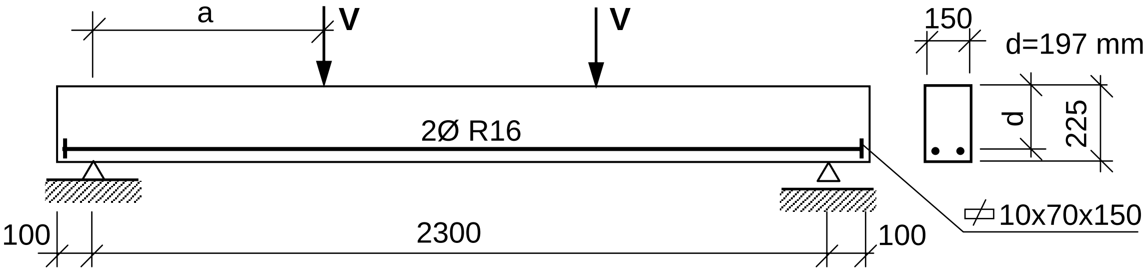

Fig. 1 Details of test beams

Tab. 1 Properties of concrete by [1]

Series	Mixture proportions	w/c	V_f (%)	f_c (MPa)	f_{sp} (MPa)	Tensile strength	
						Ultimate (MPa)	Residual (MPa)
A	1:2.80:2.60	0.62	0	24.2	2.61	2.14	0.00
B			0.05	29.1	2.67	2.22	0.53
C			0.75	29.9	2.88	2.30	0.68
D			1.00	30.0	3.36	2.56	0.83

3 Estimation of shear strength of reinforced concrete beams

3.1 Formulas for estimation of shear strength

A number of investigators have proposed analytical and empirical formulas based on tests for estimating shear strength and shear cracking stress of SFRC beams.

Using fracture mechanics principles, Bažant and Kim [2] proposed the following formula for predicting the ultimate shear strength of steel reinforced concrete beams v_u (MPa):

$$v_u = 0{,}83\xi \sqrt[3]{\rho}\left[\sqrt{f_{c,cube}} + 249{,}28\sqrt{\rho/(a/d)^5}\right], \tag{1}$$

$$\xi = 1/\sqrt{1+(d/25d_a)},$$

where $\rho = A_s/(b_w h)$ is the tensile reinforcement ratio, A_s (mm^2) is the area of longitudinal tension reinforcement, b_w, h, d (mm) is the width, overall and effective depth of the beam, $f_{c,cube}$ (MPa) is the concrete cube compressive strength, a (mm) is the shear span, d_a (mm) is the maximum aggregate size, ξ is the aggregate size effect factor.

Sharma [3] proposed a simple empirical formula for prediction of the shear strength of SFRC beams

$$v_u = kf_{ct,sp}\left(d/a\right)^{0,25}, \tag{2}$$

where $k = 2/3$ is coefficient obtained from tests, $f_{ct,sp}$ (MPa) is the concrete splitting strength, $f_{ct,sp}=0.79\ (f_{c,cyl})^{0.5}$ when the tensile strength is unknown, $f_{c,cyl}$ (MPa) is the concrete cylinder compressive strength. Other parameters are defined in formula (1).

Narayanan a Darwish [4] proposed the following formulas for prediction of the average ultimate shear stress v_u (MPa) of SFRC beams at shear failure:

$$v_u = \left(e\left[0.24 f_{fct,sp} + 80\rho \frac{d}{a} \right] + v_b \right),\ e = \begin{cases} 1.0 & ak \quad a/d > 2.8 \\ (2.8d/a) & ak \quad a/d \le 2.8 \end{cases},\ v_b = 0.41\tau F \qquad (4)$$

and shear cracking stress v_{cr} (MPa)

$$v_{cr} = (0.24 f_{fct,sp} + 20\rho d/a + 0.5F) \qquad (3a)$$

$$v_{cr} = 3\left(f_{fct,sp}\right)^{2/3}\left(\rho d/a\right)^{1/3}, \qquad (3b)$$

where $f_{fct,sp}$, $f_{c,cyl}$ (MPa) is the splitting and cylinder compressive strength of fiber concrete, $F = (L_f/D_f)\ V_f\, d_f$ is fiber factor, L_f, D_f (mm) is the fiber length and diameter, V_f (%) is the fiber volume fraction, $d_f = 0.5$ for round fibers, 0.75 for crimped fibers, and 1.0 for hooked fibers, τ (MPa) is average fiber matrix bond stress, taken as *4.15 MPa* (based on recommendation of Swamy, Mangat and Rao [5]). Remaining parameters are defined in formula (1).

Ashour, Hasanain a Wafa [6] tested 18 SFRC beams. Based on these results, they proposed two formulas for predicting shear strength of beams v_u (MPa):

a) modified ACI [7]: $v_u = \left[\left(0.7\sqrt{f_{fc,cyl}} + 7F\right)\left(d/a\right) + 17.2\rho d/a \right],$ (5)

b) modified Zsutty [8]:

$$v_u = \left[\left(2.11\sqrt[3]{f_{fc,cyl}} + 7F\right)\left(\rho d/a\right)^{0.333} 2.5/\left(a/d\right) + v_b\left(2.5 - a/d\right) \right] \quad \text{for a/d} \le 2.5 \quad (6b)$$

$$v_u = \left(2.11\sqrt[3]{f_{fc,cyl}} + 7F\right)\left(\rho d/a\right)^{0.333},\ v_b = 1.7F \quad \text{for a/d} > 2.5 \quad (6a)$$

All parameters are defined in formula (1).

Based on results of their own tests, Imam, Vandewalle and Mortelmans [9] modified Bažant's and Kim's formula and proposed two formulas for prediction of the shear strength of SFRC beams:

$$v_u = 0.83\xi\sqrt[3]{\rho}\left[f_{fct,sp} + 740\sqrt{\rho/\left(a/d\right)^5} \right], \qquad (7)$$

$$v_u = 0.7\xi\sqrt[3]{\rho}\left[f_{fc,cyl}^{\ 0.44}(1+F^{0,33}) + 870\sqrt{\rho/\left(a/d\right)^5} \right], \qquad (8)$$

where $d_f = 0.5$ for round fibers, *0.9* for crimped fibers, and *1.0* for hooked fibers. Other parameters are defined in formulas (1) and (4).

Based on Zsutty's formula [8] and results of their tests, Kwak et al. [10] proposed the formula for prediction of the shear strength of SFRC beams:

$$v_u = \left(2.1 e f_{fct,sp}^{0.7} \left(\rho \frac{d}{a} \right)^{0.22} + 0.8 v_b^{0.97} \right),\ e = \begin{cases} 1.0 & ak \quad a/d > 3.5 \\ (3.5d/a) & ak \quad a/d \le 3.5 \end{cases},\ v_b = 0.41\tau F \qquad (9)$$

All parameters are defined in formulas (1) and (4).

3.2 Numerical approach based on the finite elements method (FEM)

Two specific structural analysis programs, Lusas and Atena, based on finite elements method were used for predicting the shear crack stress and strength beams. 2-D elements, which are used, had quadrilateral shape with quadratic interpolation because these elements give theoretically the most accurate results. Concrete strengths were given by Tab. 1, elastic modulus and yield stress of steel bars were given by the test. In Lusas, multi-crack concrete model (plastic-damage-contact model) was used. In Atena, material model SBETA (an embedded fixed crack model) was used. Bond between concrete and bars was perfectly.

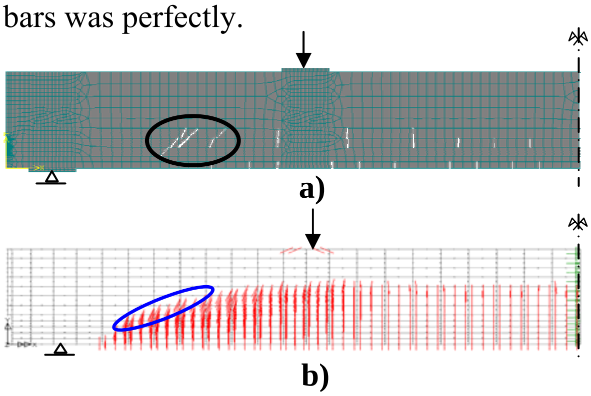

Fig. 4 Initiation of shear cracks: a) in the Atena, b) in the Lusas

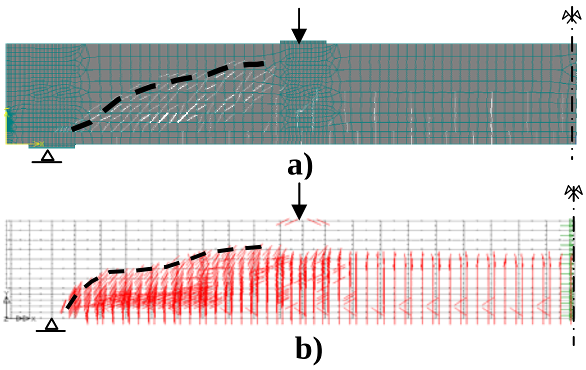

Fig. 5 Crack patterns at maximum load: a) in the Atena, b) in the Lusas

In Atena, shear crack force was defined by means of loading increment at which appeared first inclined crack (Fig. 4a). In Lusas, the shear crack force was defined by means of loading level at which orientation of flexural crack initiate to change (Fig. 4b).

4 Evaluation of formulas, experimental results and results of FEM simulation

As evident, the shear resistance V_u increases with the increasing V_f and decreasing *a/d* ratio. At *a/d = 2.0*, results obtained from the tests and the numerical simulation showed that beams failed in shear. The best correspondence with the tests was obtained by formulas (4) a (6). The results obtained from the other formulas differed from the tests for more than *40 %*. At *a/d = 4.4*, the beams failed dominantly in flexure. The most accurate results were obtained from formulas (2) and (6). Formula (8) gives relatively exact results, but formula (9) is not proper at all. Formulas (4) and (5) are not proper for beams with V_f higher than *0.5 %*.

The shear cracking force increases with the increasing V_f. Formulas (3a) and (3b), proposed by Narayanan and Darwish [4], give relatively exact results in comparison with the tests for *a/d = 2.0* and, moreover, the formula (3b) gives more accurate results than the formula (3a). At *a/d = 4.4*, both formulas are proper only for beams with V_f lower than *0.5 %*. For V_f higher than *0.5 %*, they give higher greater values than the test results for about *10~30 %*.

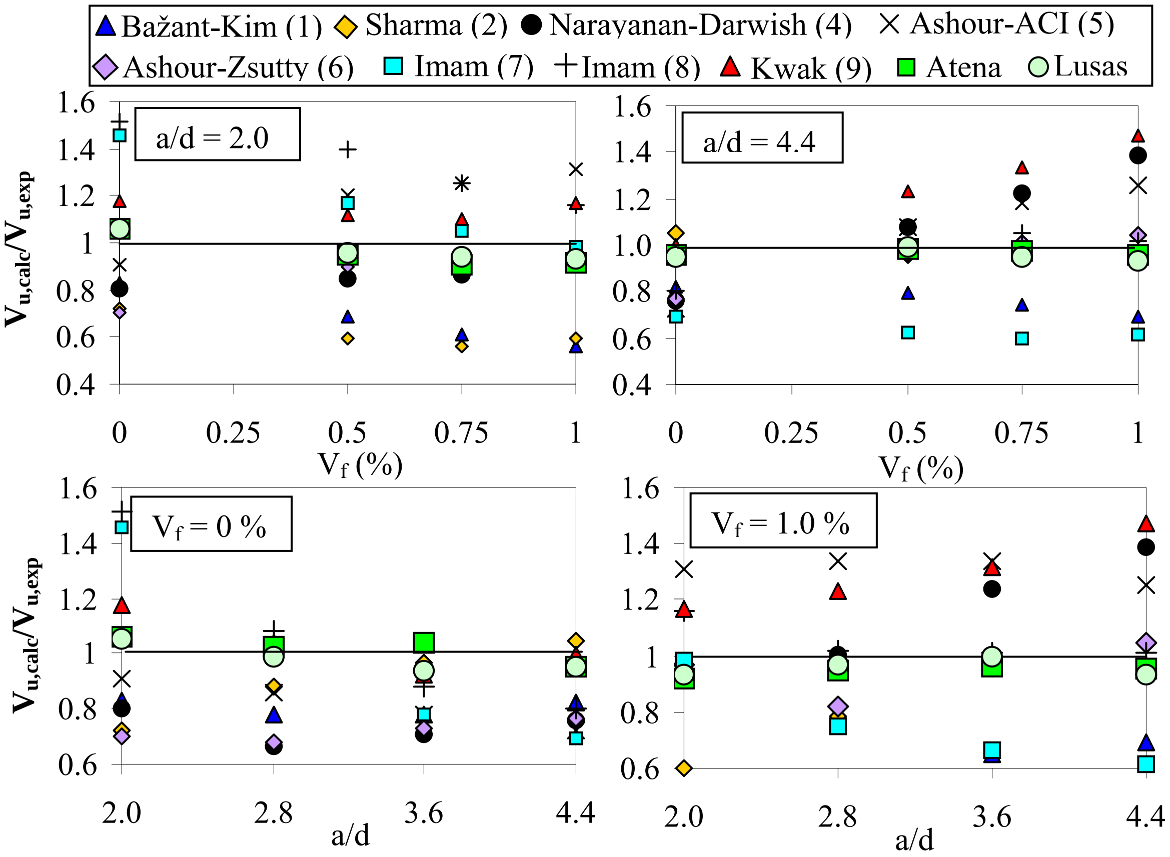

Fig. 6 Calculated to experimental ultimate shear force ratio at various ration *a/d* and V_f

From the point of view of reliability, the accepted values must be lower than *1.0*. From Figs. 6, it is evident that the formula (4) gives the most accurate results for *a/d = 2.0* and *2.8*, however, it loses accuracy for *a/d = 3.8* and *4.4*. For *a/d = 4.4*, the formula (6) proposed by Ashour [9], gives the results corresponding best with the tests.

For prediction of the shear cracking force, both formulas (3a) and (3b) can be considered to be proper for all investigated *a/d* ratios, however, only for beams with V_f lower than *0.5 %*. For beams with V_f higher than *0.5 %* and ratios *a/d = 3.6* and *4.4,* both formulas (3a) and (3b) are not proper for predicting the shear cracking force.

The numerical simulation by means of the FEM corresponds well with the test results in all case. The results obtained from Atena were higher than the tests results only for the shear-cracking force for at *a/d = 2.0*. The inaccuracy in this case may be related to uncertainty in determination of the load level at which first shear crack was created.

3 Conclusions

The formulas used for predicting ultimate shear and shear cracking force did not give a good agreement with the test results in most investigated cases. They mostly give higher values than the test results because the authors of the presented formulas might have overestimated the influence of some investigated parameters, mainly the influence of fibers. Another factor causing the difference in results could be that the empirical formulas were proposed on own tests of the individual authors, carried out in specific test conditions with different experimental procedures and ways of result processing. This factor should be eliminated or minimized, e. g. using a numerical simulation with a larger range of investigated parameters.

Some of the formulas, e. g. (2), (4), (7) and (9), express the shear strength by using the concrete splitting strength, while others, e. g. (1), (5), (6) and (8), by using the concrete compressive strength. To express the influence of fibers on shear strength it would be more accurate to use the splitting strength than the compressive strength, because fibers almost do not increase concrete compressive strength, but clearly increase concrete tensile strength. In spite of this, the most accurate formula is (6) proposed by Ashour [6], which gives results max. *20 %* lower than the experimental ones.

Results obtained from the numerical simulation give good agreement with test results, so the above presented computational models could be used for a proper simulation of the behaviour of concrete beams subject to shear force and predicting their shear resistance or shear cracking force.

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