CONCRETE-REINFORCEMENT INTERACTION MODELLED BY THE FETI METHOD

Jaroslav Kruis[1], Petr Štemberk[2]

Abstract

Interaction between concrete and reinforcement is very important phenomenon which is studied continuously. Unfortunately, there are some numerical difficulties which can be remedied by techniques developed for the FETI method. This contribution concentrates on numerical approach. Material laws are not studied.

Keywords: mperfect Bond, Concrete-reinforcement Interaction, Pull-out Test, FETI Method, Parallel Computing

1 INTRODUCTION

Reinforced concrete is a common material used in civil engineering. Interaction between concrete mass and reinforcement plays an important role in design and the interaction has to be taken into account. Simple design methods are based on an assumption of perfect bonding between the two constituents and there are provisions in national codes as well as in Eurocodes dealing with necessary length of the reinforcement in order to activate the perfect bond.More sofisticated methods take into account the interaction between concrete and reinforcement. This is generally a complicated problem and it should be solved numericaly. There are basically two groups of methods. The primal methods which deal with the primal variables, usually unknown displacements, and the dual methods which eliminate the primal unknowns and deal with the dual unknowns, usually forces. The primal methods are straightforward while the dual methods require some time to get familiar with. Short description of the dual methods follows. For simplicity, let us assume only one steel bar in concrete mass. There is an interface between them and in the case of perfect bond there are the same displacements on the interface. The same displacements can be enforced by Lagrange multipliers which play role of nodal forces or stresses with respect to discrete or continous formulation. The displacements can be eliminated from the problem and only the Lagrange multipliers remain. This is very similar to the FETI (Finite Element Tearing and Interconnecting) method which was introduced in 1991 by Farhat and Roux [1]. The FETI method is a domain decomposition method and it is useful for solution of large problems especially on parallel computers.

[1] CTU in Prague - Faculty of Civil Engineering, Department of Mechanics, Thákurova 7,166 29 Prague 6, Czech Republic, e-mail: jk@cml.fsv.cvut.cz

[2] CTU in Prague - Faculty of Civil Engineering, Department of Concrete and Masonry Structures, Thákurova 7, 166 29 Prague 6, Czech Republic, e-mail: stemberk@fsv.cvut.cz

2 MODIFICATION OF THE FETI METHOD

With respect to the limited number of pages in this contribution, detailed formulation of the FETI method will not be described here and reader is referred to references [2], [4], [3] among many others. The FETI method is a nonoverlapping domain decomposition method, where the original domain is split into several smaller subdomains. The subdomains are treated independently and Lagrange multipliers are defined on subdomain boundaries in order to enforce continuity. After some manipulation, the coarse system of equations is obtained which contains only unknown Lagrange multipliers and a very limited numberof additional coefficients. The classical FETI method enforces the same displacements on subdomain boundaries. If there is a reason for different displacements between two neighbour subdomains, the continuity condition transforms itself to a slip condition. Let the boundary be split to two disjunct parts. The continuity condition is valid on the first part of the boundary and has the form

$$\boldsymbol{B}_c \boldsymbol{u} = \boldsymbol{0}, \tag{1}$$

where $\boldsymbol{B}_c$ denotes a signed Boolean matrix and the vector $\boldsymbol{u}$ contains unknown nodal displacements. The slip condition is valid on the second part of the boundary and has the form

$$\boldsymbol{B}_s \boldsymbol{u} = \boldsymbol{s}, \tag{2}$$

where $\boldsymbol{B}_s$ denotes a signed Boolean matrix and the vector $\boldsymbol{s}$ contains slip components. The conditions (1) and (2) can be merged to a new interface condition

$$\boldsymbol{B}\boldsymbol{u} = \begin{pmatrix} \boldsymbol{B}_c \\ \boldsymbol{B}_s \end{pmatrix} \boldsymbol{u} = \begin{pmatrix} \boldsymbol{0} \\ \boldsymbol{s} \end{pmatrix} = \boldsymbol{c}. \tag{3}$$

The energy functional can be written in the discrete form

$$\Pi = \frac{1}{2}\boldsymbol{u}^T \boldsymbol{K} \boldsymbol{u} - \boldsymbol{u}^T \boldsymbol{f} + \boldsymbol{\lambda}^T (\boldsymbol{B}\boldsymbol{u} - \boldsymbol{c}), \tag{4}$$

where K denotes the stiffness matrix, f denotes the vector of nodal forces and denotes the vector of Lagrange multipliers. The stationary conditions havethe form

$$\boldsymbol{K}\boldsymbol{u} - \boldsymbol{f} + \boldsymbol{B}^T \boldsymbol{\lambda} = 0, \tag{5}$$
$$\boldsymbol{B}\boldsymbol{u} = \boldsymbol{c}. \tag{6}$$

The system of two stationary conditions has to be accompanied by the solvability condition

$$\left(\boldsymbol{f} - \boldsymbol{B}^T \boldsymbol{\lambda}\right) \perp \ker \boldsymbol{K} = \boldsymbol{R}, \tag{7}$$

where the matrix R contains rigid body motions of the floating subdomains as its columns. The unknown vector u can be expressed from the equation (5) in the form

$$\boldsymbol{u} = \boldsymbol{K}^+ \left(\boldsymbol{f} - \boldsymbol{B}^T \boldsymbol{\lambda}\right) + \boldsymbol{R}\boldsymbol{\alpha}, \tag{8}$$

where the pseudoinverse matrix $\boldsymbol{K}^+$ has to be used with respect to floating subdomains and the vector α contains coefficients of linear combinations of the rigid body motions. The interface conditions has the form

$$BK^+B^T\lambda = BK^+f + BR\alpha - c \tag{9}$$

and the solvability condition has the form

$$R^T\left(f - B^T\lambda\right) = \mathbf{0}. \tag{10}$$

Usual notation in the FETI method is the following

$$\begin{aligned} F &= BK^+B^T, & (11)\\ G &= -BR, & (12)\\ d &= BK^+f, & (13)\\ e &= -R^Tf. & (14) \end{aligned}$$

The coarse problem can be written with the help of notation (11) - (14) in the form

$$\begin{pmatrix} F & G \\ G^T & \mathbf{0} \end{pmatrix} \begin{pmatrix} \lambda \\ \alpha \end{pmatrix} = \begin{pmatrix} d - c \\ e \end{pmatrix}. \tag{15}$$

The modified coarse problem (15) differs from the coarse problem of the classical FETI method by the vector of prescribed slips c on the right-hand side.

The prescribed slip between two subdomains is not a common case. On theother hand, the slip often depends on shear stress. Discretized form of equationsused in the coarse problem requires a discretized law between slip as a differenceof two neighbour displacements and nodal forces as integrals of stresses alongelement edges or surfaces. One of the simplest law is the linear relationship

$$c = H\lambda, \tag{16}$$

where $\boldsymbol{H}$ denotes the compliance matrix. Substitution of (16) to the coarse problem (15) leads to the form

$$\begin{pmatrix} F + H & G \\ G^T & \mathbf{0} \end{pmatrix} \begin{pmatrix} \lambda \\ \alpha \end{pmatrix} = \begin{pmatrix} d \\ e \end{pmatrix}. \tag{17}$$

It should be noted that the coarse system of equations (17) is usually solved bythe modified conjugate gradient method. Details can be found in references[2,3].The only difference with respect to the coarse system of the classical FETI method is the compliance matrix H. Only one step, the matrix-vector multiplication, of the modified conjugate gradient method should be changed. Thecompliance matrix may be a diagonal or nearly diagonal matrix.

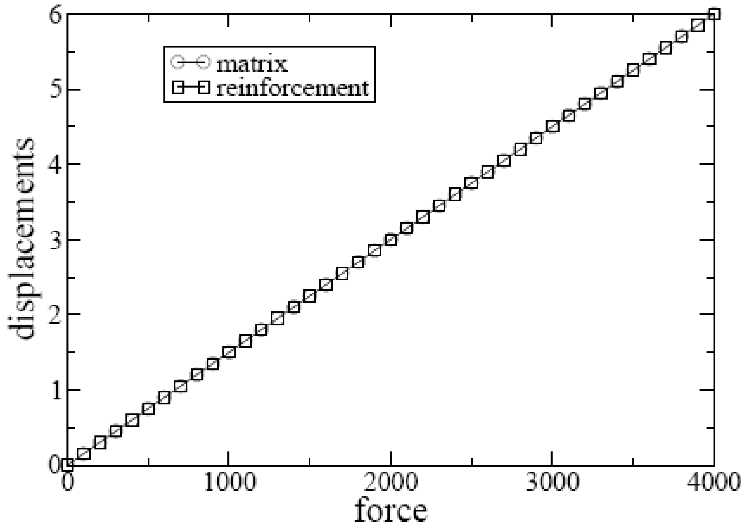

Figure 1: Perfect bond.

3 NUMERICAL EXAMPLES

Capabilities of the proposed approach are demonstrated on a simple example Concrete mass contains one steel bar which is loaded by tensile force. This is the so-called pull-out test. Three different regimes of interaction between concrete and steel bar are presented. Graphical output is obtained for two selected points, one on the steel bar and the other on concrete mass. The selected points are identical before load application. The first example shows the perfect bond between the concrete and steel. Figure 1 shows displacements on the vertical axis and the applied force on the horizontal axis. In the case of the perfect bond, both lines are identical as well as displacements. The second example shows imperfect bond, where any force causes slip between reinforcement and the concrete mass. The situation is depicted in Figure 2. There are two lines, where the upper line shows displacements of steel bar while the lower line shows displacements of concrete mass. Finally, the third example shows imperfekt bond with some delay. It means, up to some magnitude of the applied force the bond is perfect while beyond this magnitude an imperfect slip occurs. In the first part of loading, the lines are identical and therefore the displacements are the same, while in the second part of the loading there is a slip between the steel bar and the concrete mass. The situation is depicted in Figure 3.

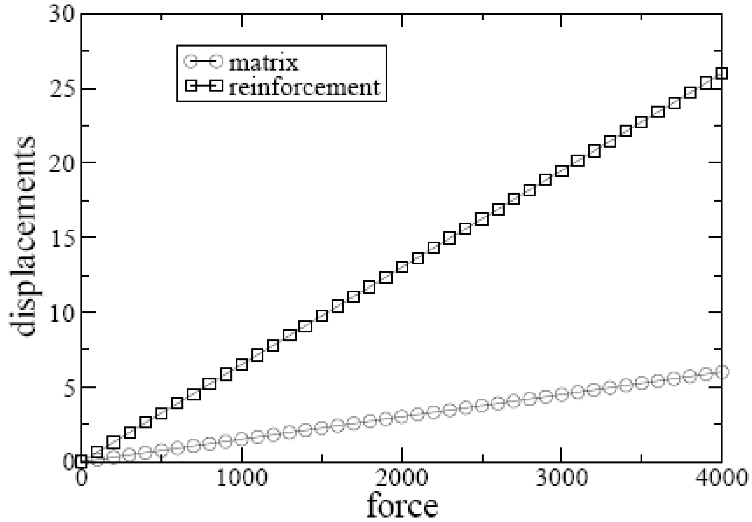

Figure 2: Imperfect bond.

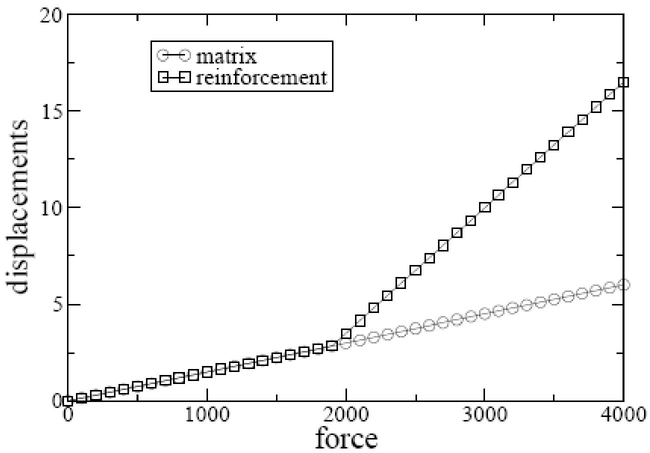

Figure 3: Imperfect bonding with delay.

4 CONCLUSIONS

This contribution deals with modelling of bond between reinforcement (steel bar) and composite matrix (concrete mass) from a numerical point of view. Small modification of the FETI method is used and promising results are obtained. The proposed strategy is applicable also for large problems with many pieces of reinforcement because the FETI method can be easily parallelized.

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