MODELLING THE BEHAVIOUR OF TEXTILE REINFORCED CEMENTITIOUS COMPOSITES UNDER BENDING.

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Abstract

Textile reinforced cementitious composites with glass fibres as reinforcement exhibit relatively high strength and ductility and thus provide and interesting new material for construction purposes. Freeform architecture and lightweight construction elements only represent a small part of the construction types in which the advantages of this material could be exploited. The material itself is characterised by a linear behaviour in compression and a non linear tensile behaviour. Over the years many efforts have been directed into the modelling of this tensile behaviour. However, to be able to design structures one not only has to be able to predict the behaviour under compression and tension but also under bending. The introduction of inter-laminar shear stresses might lead to debonding and thus might limit the stiffness and strength of the studied material in bending. In this paper the predicted load displacement curves, based on calibrated material models in compression and tension, will be compared with experimental curves in order to detect possible delaminating effects.

Keywords: textile reinforced cementitious composites; glass fibres; inter-laminar shear; bending; delaminating effects

1 Introduction

In order to commercially utilize new composite materials in civil engineering applications, simple and effective analysis and design guides are needed. The materials are designed to improve the performance of constructions. Textile reinforced cementitious composites (TRC) with glass fibre reinforcement are new composite materials which can be tailored to the needs and specifications of the constructive elements. By using a fibre volume fraction which is exceeding the critical fibre volume fraction, the fibres can ensure strength and stiffness at applied loads far exceeding the range in which matrix multiple cracking occurs. The TRC composites are interesting new materials exhibiting a high strength and ductility. If however we want to design and optimize a construction with a new material the development of a material model is necessary. Over the years much effort has been directed into the modelling of the non linear tensile behaviour. However, to be able to design structures one not only has to be able to predict the behaviour under compression and tension but also include effects which typically could occur only under bending:

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limited inter-laminar shear strength might lead to debonding and thus might limit the stiffness and strength of the studied material in bending. The main focus in this paper will thus be put on the bending behaviour of TRC.

2 TRC in tension and compression

As was already mentioned, knowledge of the behaviour of TRC in compression and tension will be used to predict the behaviour in bending. In compression, TRC exhibits a linear elastic behaviour up to failure. Since the use of fibre reinforcement in the form of textiles allows the introduction of a relatively high fibre volume fraction, a distinct strain hardening behaviour can be obtained in tension. In this paper the stress – strain behaviour in tension will be modelled using well known a simple ACK theory [1][3]. According to the ACK theory, three distinct stages can be detected in the stress-strain curve. “figure 1” illustrates a experimental stress-strain curve with three distinct stages (used in the ACK theory): linear elastic stage (stage I), multiple cracking stage (stage II) and post cracking stage (stage III).

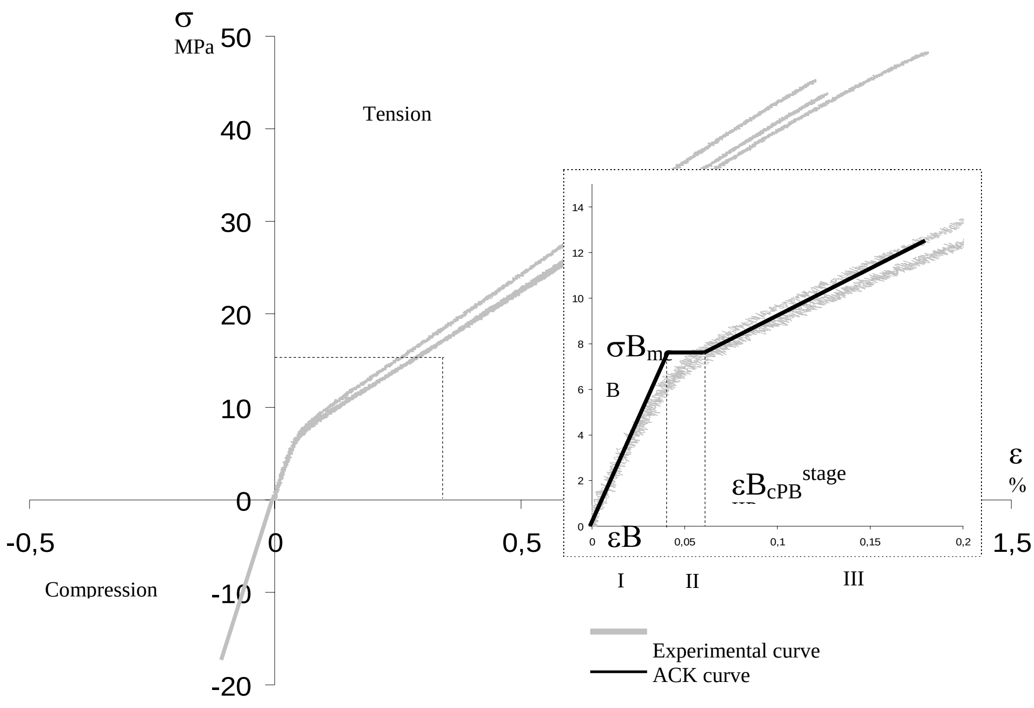

Fig. 1 experimental and theoretical stress-strain curve, according to the ACK theory (Aveston et al., 1971)

In the linear elastic stage (stage I), the stiffness of the composite E_{c1} is function of the effective fibre volume fraction V_f^*, the volume fraction of the matrix V_m, the stiffness of the fibres E_f and the stiffness of the matrix E_m. The matrix-fibre interface bond is assumed to be elastic and the composite stiffness E_{c1} can be determined by the law of mixtures:

$$E_{c1} = E_f V_f^* + E_m V_m \tag{1}$$

The effective fibre volume fraction can be calculated by using the following equation:

$$V_f^* = V_f \cdot \eta \tag{2}$$

In this equation V_f is the fibre volume fraction and η is the efficiency factor for the fibre reinforcement. At a certain unique composite stress σ_{mc}, multiple parallel cracks are introduced in the matrix. When the first crack appears and reaches a fibre, debonding of the matrix-fibre interface occurs and further matrix-fibre interaction occurs through friction. After multiple cracking, the strain of the composite $\varepsilon_c^{stageII}$ can be calculated as follows as function of the matrix failure strain:

$$\varepsilon_c^{stageII} = (1 + 0.66\alpha)\varepsilon_{mu} \tag{3}$$

$$\alpha = \frac{E_m V_m}{E_f V_f^*} \tag{4}$$

The composite multiple cracking stress σ_{mc} can be calculated from knowledge of the matrix failure stress σ_{mu} as follows:

$$\sigma_{mc} = \frac{\sigma_{mu} E_{c1}}{E_m} \tag{5}$$

Once full multiple cracking has fully occurred, only the fibres further dedicate to the stiffness in stage III (post-cracking stage). The stiffness of the composite E_c in this stage is thus:

$$E_c = E_f V_f^* \tag{6}$$

3 TRC beam element in bending

3.1 Theory

Since the behaviour of TRC in tension shows three stages, three expressions are needed to define the equilibrium of forces and moments. [2][4]By expressing the equilibrium of forces and moments, the position of the neutral axis (which will be further depicted as a) and the maximum occurring tensile strain (ε_t) in the section can be calculated for each cross section along the beam. Subsequent integration of the bending stiffness for each differential beam element with height h and a unit width, over the total length, will obtain a force displacement diagram. For each cross section the one of the following sets of equilibrium equations can be used depending on the value of the maximum tensile stress.

(1) As long as composite behaves linear elastic along the whole beam section the following expressions can be used, with M_e defined as external moment:

$$a = h/2 \tag{7}$$

$$M_e = \frac{2}{3} a^2 E_{c1} \varepsilon_t \tag{8}$$

(2) Once the composite maximum tensile strain is situated in the multiple cracking stage the equilibrium equations can be based on the internal strains and stresses in figure 2.

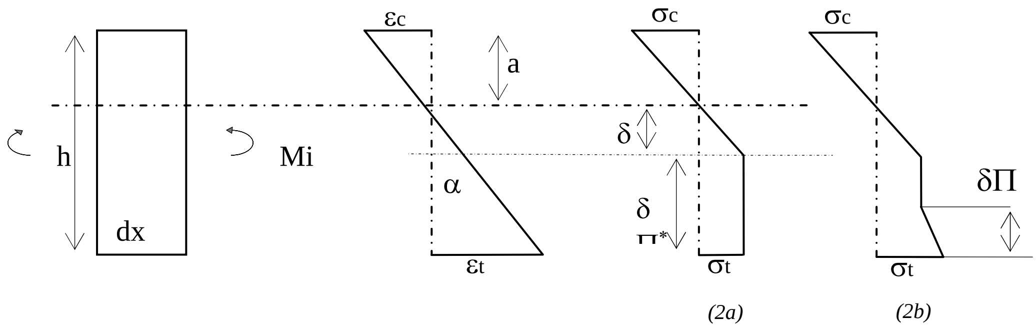

Fig. 2 distribution of stress and strain over cross section
(2a) distribution of stress strain over cross section in the multiple cracking stage
(2b) distribution of stress strain over cross section in the post cracking stage

$$E_{c1}\varepsilon_t \frac{a^2}{2(h-a)} = \delta\frac{\sigma_{mc}}{2} + \delta^*\sigma_{mc} \text{ with } \delta = \frac{\varepsilon_{mu}}{\varepsilon_t}(h-a) \text{ and } \delta^* = h-a-\delta \qquad (9)$$

$$E_{c1}\varepsilon_t \frac{a^3}{3(h-a)} + \delta^2\frac{\sigma_{mc}}{3} + \sigma_{mc}\delta^*\left(\delta + \frac{\delta^*}{2}\right) = M_e \qquad (10)$$

Finally the composite maximum tensile strain will be situated in the post-cracking stage:

$$E_{c1}\varepsilon_t \frac{a^2}{2(h-a)} = \delta\frac{\sigma_{mc}}{2} + \delta^*\sigma_{mc} + (\varepsilon_t - \varepsilon_c^{stageII})E_f V_f^{*}\frac{\delta^{**}}{2}$$
$$\text{with } \delta^{**} = h - a - \frac{\varepsilon_c^{stageII}}{\varepsilon_{mu}}\delta \qquad (11)$$

$$E_{c1}\varepsilon_t \frac{a^3}{3(h-a)} + \delta^2\frac{\sigma_{mc}}{3} + \sigma_{mc}\delta^*\left(\delta + \frac{\delta^*}{2}\right) +$$
$$\left(h - a - \frac{\delta^{**}}{3}\right)\frac{\delta^{**}}{2}(\varepsilon_t - \varepsilon_c^{stageII})E_f V_f^* = M_e \qquad (12)$$

Once the equilibriums of all sections are established, the deflection in any section can be determined by double integration of $\frac{M_e}{EI_{section}}$ along the length of the beam. $EI_{section}$ is the bending stiffness as calculated for each section of the beam using the calculated stress and strain profile determined from the equitation's of equilibrium as depicted above.

3.2 Experimental Program

In this paper two experimental series were carried out, a tensile test was used to characterise the stress strain behaviour of the specimens. The data obtained from this tensile test was then used in the proposed analytical model to predict the bending behaviour of the specimens. These predictions were then compared with experimental force-displacement curves obtained under four point bending.

3.2.1 Specimens

The specimens used in this study were glass fibre- reinforced cementitious laminates. The cementitious matrix used for all the specimens is an Inorganic Phosphate Cement (IPC). IPC has been developed at the "Vrije Universiteit Brussel" and shows a neutral pH after hardening. By using IPC it is possible to use E-glass without durability problems of the fibre reinforment. The material properties of hardened IPC are similar to those of cement-based materials. Although the strength in compression is rather high, the tensile strength is low. One standard IPC mixture is chosen in this work, without use of any fillers or retarding or accelerating components. The E-glass fibre reinforcements used in the specimens was chopped glass fibre mats (“2D-random”) with a fibre density of 300g/m² (Owens Corning M705-300). IPC laminates are cured in ambient conditions for 24 hours. Post-curing is performed at 60°C for 24 hours.

3.2.2 Tensile test

The thickness of each laminate was measured and de volume fraction was calculated.

The tensile test was carried out to obtain the matrix and fibre stiffness of the different laminates in the different stages. The stress strain curve was generated using a tensile testing machine with a capacity of 100 kN. The rate of cross head displacement was set at 1 mm/min. The strain was measured with an extensometer. For the different laminates the results of the tensile test were plotted and the parameters needed in the theoretical model are depicted in table 1.

Number of layers	Thickness (mm)	σ_{mu} (MPa)	$E_f V_f^*$ (GPa)	$E_m V_m$ (GPa)	V_f (%)	E_{c1} (GPa)
4	2.9	9.5	3.1	15.1	16.2	10.5
6	4.2	8	3.3	14.9	16.8	14.6
8	5.1	6	3.1	14.7	16.2	10.5
10	7.6	6	3.0	15.2	15.6	10.8
12	8.4	9.5	3.5	15.0	16.2	10.5
14	10.2	8.5	3.9	15.1	16.2	9.1

Table 1.: results of the tensile tests

3.2.3 Bending test

The laminates were loaded in a 4 point bending test. Two supports were placed with a span of 200 mm, the cross head was equipped with horizontal bars were two cylinder were mounted with a distance of 100 mm. The testing machine was displacement controlled; the loading rate was set to 1 mm/min. The results of the bending test were plotted in force deflection curves. Comparison of the experimental data with the numerical model has been carried out; two examples are given in the figures below. In figure 3 curves for 6 and 12 layers are depicted.

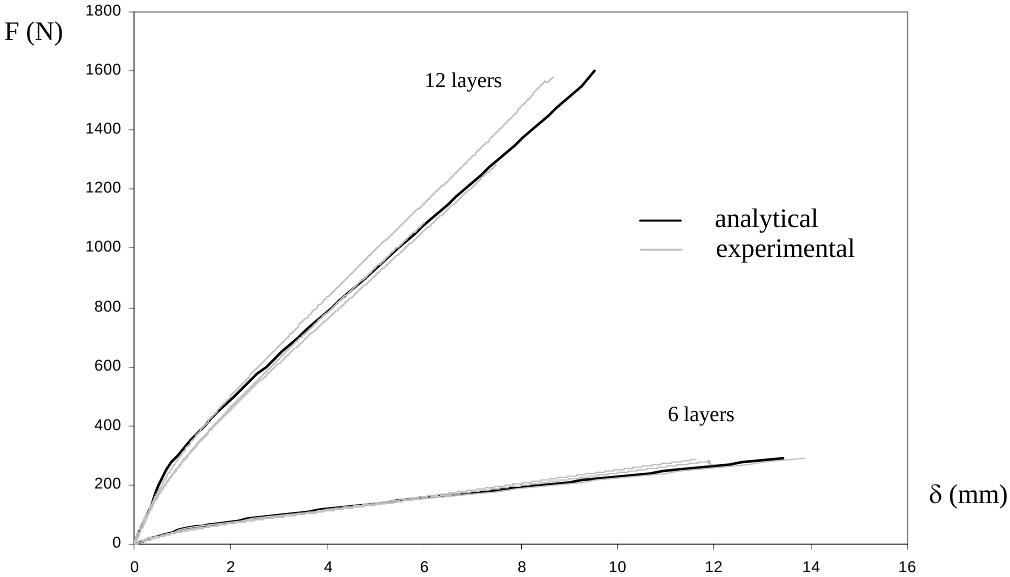

Fig. 3 comparison of experimental and analytical load deflection curves

The comparison of the experimental and the numerical revealed a non linear behaviour in both cases, with a great similarity. None of the tested specimens failed below the predicted load. Thus we can conclude for the tested series the influence of inter-laminar shear failure in can be neglected.

4 Conclusions

In this paper a method is proposed to predict the response of a TRC laminate in bending, by using the model parameters obtain form a tensile test. The stress – strain behaviour in tension was modelled using the well known simple ACK theory. The calibrated model was then used to predict the behaviour under bending of specimens with various thicknesses. Comparison with experimental data showed that there was no effect of inter-laminar shear that lead to limitation of stiffness and strength which was the main concern.

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