MODEL OF BEHAVIOR AT PULLING OF THE REINFORCED CONCRETE OF WAVED FIBERS

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Abstract:

The modeling of the behavior in pulling of the reinforced concrete of waved fibers (to variable geometry), is based on the study of the behavior of the fiber in the concrete matrix. Writing the energy equilibrium of an undulating fiber at the time of the sliding, the movement is governed by a differential equation. The pull-off strength of the fiber is obtained by integrating this one on the length anchored of the fiber, the phenomenon of friction fiber-concrete matrix, the influence of the radial constraints (due to the undulation) and the case of the possible plastification of the steel constituting the fiber, one considered in the modeling. A solution to the problem of the modeling of the adhesion waved fiber - concrete matrix was proposed bringing to light the evolution of the constraint of adhesion (τu) according to the "t" anchoring. This modeling is introduced in an effort relation-opening of the cracks, and then a passage to the law (constraint –deformation) was made by modifying this relation. Finally for every modeled parameter (adhesion, constrained-deformation in pulling and pickets submitted to a compound flexion), a calculation confrontation/essay was been executed. The results of the confrontation are rather satisfactory.

Keywords: Concrete fiber, Fibers corrugated (waved), Adhesive, Friction, Modeling

1 Introduction

The behavior of the composite (concrete fiber) is elastic linear until cracking of the matrix. The interior studies that one can quote [1], [2] and [5] show that at the instant where appears a crack there are a brutal decrease of the constraint. We interest us in that follows to the behavior after cracking, therefore after the peak of load. In this domain, one considers the sliding phenomenon of fiber-matrix eventual plasticization of the fiber steel. One expresses then the energizing equilibrium of a fiber at the time of the sliding.

2 Modeling of the adhesion matrix in concrete Caption

2.1 Equations of static equilibrium for an element of wire-drawn fiber

The wire-drawn fiber (said also to variable geometry) is decomposed in curves sections of curvilinear elementary length “ds” (see Fig. 1). The equilibrium can be written in the following manner [Eq (1 to 4). The projection of the forces along;

The x axis: $$P\,sin\left(\frac{dq}{2}\right)+\left(P+dP\right)sin\left(\frac{dq}{2}\right)=dN \quad (1)$$

The z axis: $$dP\,cos(\frac{dq}{2}) = dT \quad (2)$$

For the small angle to elementary elements «ds», we can do: $sin(\frac{dq}{2}) = \frac{dq}{2}$ and $cos(\frac{dq}{2}) = 1$

One ignoring the higher order terms $dP(\frac{dq}{2})$, the equations (1) and (2) become (3 and4) [5]

$$dN = Pdq \quad (3)$$
$$dP = dT \quad (4)$$

The equilibrium of an element of fiber at the time of the sliding, generated a radial effort dN and the tangential effort of dT, that two efforts can be linked by a friction law of coulomb, this law consists to linking the shear stress τ to the normal stress σ of a facet by a friction coefficient f and a cohesion Co. We will adopt a coulomb's law in global forces, where the equation (4) as eq (5): $$dT = f\,dN + t_o\,p\,ds \quad (5)$$

Where p is the perimeter of the fiber and τo is the bond stress between the fiber and the matrix.

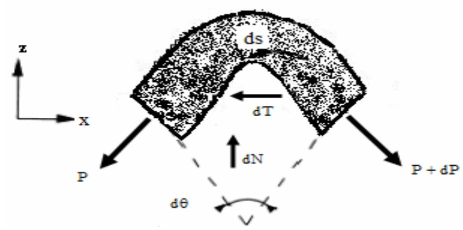


Fig. 1 Equilibrium of small element wire-drawn fiber [5]

2.2 Energy balance sheet during sliding

Physically at the time of the sliding, a fiber element undergoes, on the one part, a sliding of rigid body of amplitude dδ and on the other part, a variation of curvature (dC) produced by flexion. The fiber follows then the print of its initial geometry, the experimental observances [5] showed that each section undergoes a plasticization. The purely statics approach of the fiber equilibrium does not permit an assessment of plastic energy dissipated during the sliding of the fiber. The equilibrium of this fiber element curve must be described from an energy point of view.

The balance of the mechanical energy, applied to the slipping elementary length curve of a fiber that, allows to write that the work of the external forces (W_{ext}) is equal to the work of the deformations (W_{def}),Eq (6) $$W_{ext} = W_{def}$$ (6)

The work of the external forces written as Eq (7): $$W_{ext} = \int_S p_i\, u_i\, dS \quad (7)$$

Correspond to the integration, of the product of the elementary forces pi .dS, over the external surface of the fiber element's, (pi is the pressure exerted on the exterior surface u_i are the kinematic displacement field of the element and i is the indication by representing a coordinate in the space. The strengths of mass, such as the gravity, are disregarded 5.In the field of the displacements considered, only the components of the efforts (P, P+dP and dT) hard working follow the curvilinear axis is taking in account (fig 01). Disregarding the terms of higher order, the equation (7) written as eq (8) :

$$W_{ext} = -P\,dd + (P + dP)dd - dT\,dd \leftrightarrow W_{ext} = dP\,dd - dT\,dd \qquad (8)$$

The work of the internal efforts is obtained by integration, over the element's volume V, of the product of the stress tensor σ_{ij} and strain tensor ε_{ij} defined in all dawned of the element's volume eqt (9). $W_{def} = \int_V s_{ij}\, e_{ij}\, dV = \int_V s\; z\, dC\, dV = dC \int_V s\; z\, dV$

$$W_{def} = dC\; ds \int_S z\; s\; dS \; = dC\; ds\; M \qquad (9)$$

With: **M**: bending moment in the fiber, **ds**: elementary curvilinear length of the fiber.
dS: straight section of the element's fiber.

Considering the relations (8) and (9), the balance of the mechanical energy can be written Eq.(10): $dP\,dd - dT\,dd = M\,dC\,ds$ (10)

Combining the equations (3), (5) and (10), we obtaining the equation (11).

$$dP = (P\; f\; C + t_o\, p + M\, C'\,)\, ds \qquad (11)$$

Where C' is the first derivative of the curvature C against the curvilinear abscissa.

The result of the integration of the Eq (11) along the fiber gives a differential equation of the first order in P. In order to define the behavior's problem of the wire-drawn fiber in the concrete matrix, the modeling of the fiber adhesion stamps (coefficient of rubbing) and the effect of the tilting of these fibers will do one of the principal aims of this study.

2.3 Modeling of the adhesion (coefficient of rubbing)

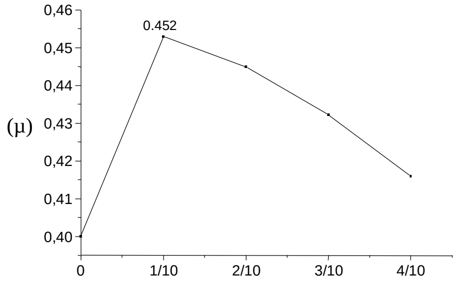
The coefficient μ is then defined starting from the isostatic equilibrium equations between the asperities in interaction (figure 2). Roughness rubbing has a sinusoidal profile, it results the following expressions. $m = (f + c)/(1 - fc)$ (12)

$$c = p\,\frac{A}{l}\cos\left[p\,\frac{d}{l}\right] \qquad (13)$$

With; d < l/**2**, l/**2**: period, **χ** : the slope of the contact, **A** : amplitude of the rough,

d : Relative displacement of the rough and **A**/l ≤ 0,015 [7].

In the equation (13), the condition $\delta < \lambda/2$ drive at $[\pi\delta/\lambda] < \pi/2 \Rightarrow \chi = 0$ from where; $[\delta/\lambda] < 1/2$. The equations (12) and (13) make it possible to plot the curve representing the evolution of μ according to the ratio $[\delta/\lambda]$ (fig 3). This one makes it possible to fix the coefficient of apparent friction. Consequently, the variation of the tangential stress T given by the equation (6) is transformed into equation (14). $dT = m\, dN + t_o\, p\, ds$ (14)



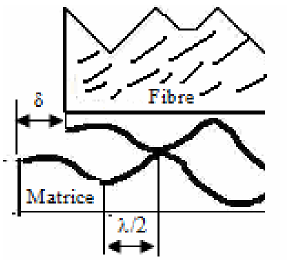


Fig. 2 Sinusoidal profile of the roughness rubbing

Fig. 3 Evolution of the visible rubbing coefficient μ according to δ /λ

2.4 Determination of the probabilistic functions density for the orientation of fibers in the matrix (case of an orientation 2D)

The fiber is tilted of an angle θ compared to the axis of traction x1 (see fig 4). This slope causes an effect of polished of friction and consequently, an increase in the forces compared to those of a fiber of null slope [3].

By referring to [7], the expression of the function of distribution G is given by the equation (15).

$$dG = (f,y)dfdy \qquad (15)$$

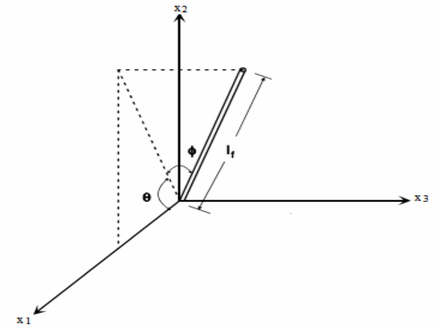


Fig. 4 Representation of an axis of waved fiber in the space

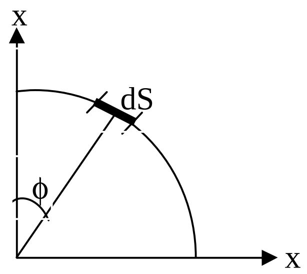


Fig. 5 Definition Space of the probabilistic density function (2D)

The fibers are directed arbitrary in the plan of the matrix concrete, in is interested only in the fibers which are in the plans of the force of pulling, in this case of orientation in 2D, (fig 5), ψ=0, the expression of g (ϕ,ψ) is transformed into g (ϕ).

The determination of g (ϕ), into fixed a space of definition S, represented by a quarter of circle of ray unit. Probability that a point of S determined by the angle of orientation ϕ is on the part of S noted ΔS is:

$$P_{rob}\left[g(f,y)\in DS\right] = \frac{DS}{S} = \int_{DS} dG \qquad (16)$$

$\int_{DS} dG = \frac{DS}{S} = dG$, With $dG = (f,y)dfdy$ and $dG = (f)df = \frac{dS}{S} = \frac{1.df}{\frac{2p.1}{4}}$

$$(f)df = \frac{2}{p}df \text{ , where } g(f) = \frac{2}{p} \qquad (17)$$

With 1: represents the unity ray.

The multiplicative factor is: $e^{\mu\theta}$, with μ coefficient of rubbing for the couple fiber matrix.

$$P_q = e^{mq} P_{q-0} \qquad (18)$$

In the system of axes (x1, x2, x3), one can write; $cos q = sin f cos y$

And we have (eqt 19); $$e^{mq} = e^{mq \, ar\,cos[sin f \; cos y \,]} \tag{19}$$

The generalization of this expression to all fibers is done by application of the operator mathematical expectation of the function g (ϕ) given in (17).

$$h = < e^{mq} > = < e^{mq \, ar\,cos[sin f \; cos y \,]} >$$

For a two-dimensional and random distribution, one has; $g(f) = \dfrac{2}{p}$

$$h = \frac{2\,e^{m\frac{p}{2}}}{m\;p}\left(1 - e^{-m\frac{p}{2}}\right) \tag{20}$$

The influence of tilted fibers is then taken into account by the parameter η equation (21)

Finally, the expression of the effort P (S) necessary to pull the wire-drawn fiber of the matrix concrete, is given by the combination of the equations (10), (14) and (20), the equation (21) obtains some after integration over the length of the fiber [4].

$$P(s) = h\,\frac{t_o p + MC'}{-mC}\left(e^{m\,C\,s} - 1\right) \tag{21}$$

3 Confrontation forces displacements curve to experimental results

The developed model makes it possible to describe the behavior of a fiber undulated during its wrenching of a concrete matrix. It makes it possible to plot the curve effort of wrenching according to displacement. The confrontation of the curves obtained by the model suggested and the curves obtained on tests of wrenching carried out by [5], for various lengths of fiber (lf = 16 and 24 mm), is given to the figures (6) and (7). The result of this confrontation is rather satisfactory. The shape of the experimental curves is correctly approximate. The model suggested is based on the application of the theorem of mechanical conservation of energy. The behavior is described using a first order differential equation. It utilizes friction with the interface matrix fiber of concrete, the radial stress modifying those of shearing and the plasticization of steel. These phenomena are brought into play during the wrenching of corrugated fiber. The geometry of fiber plays a dominating part in a test of wrenching. The fiber dissipates, during separation, an energy of friction in contact with the matrix (variable according to the intensity of the normal stress) and, at the time of the slip, an energy of plasticization result to the rectification of fiber. These two phenomena are taken into account. The behavior of the test-tubes is approximate in a rather satisfactory way.

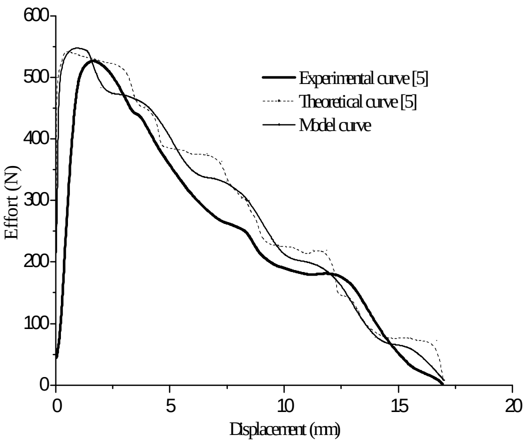


Fig. 6 Curve force – displacement (lf = 16 mm)

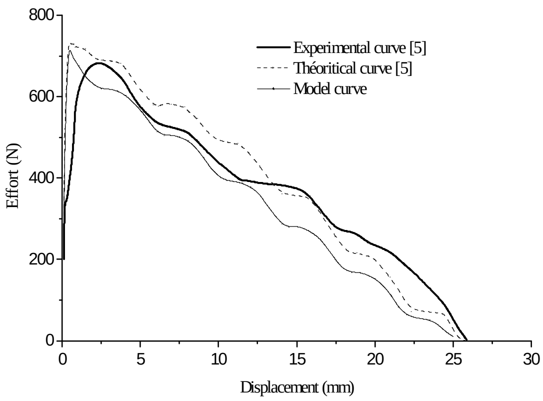


Fig. 7 Curve force – displacement (lf = 24 mm)

4 Adaptation of the relations stress deformation suggested by [1, 2] to wire-drawn fiber

4.1 Determination of the adhesion constraint (t_u) of the undulating fibers

In order to determine an average constraint of adhesion matrix fiber, the calculation is executed for the three lengths of corrugated fibers quoted previously (16 and 24 mm). This constraint is given by the expression (22).

$$t_u = \frac{P(s)}{p\ f\ \ la} \tag{22}$$

With; **P(s)**: effort of **C** rooting up the fiber, f: Diameter of the fiber,
la: adherent length of the fiber (la = lf . t /2), **t**: parameter of anchorage [1, 2].

The term (t) was the subject of a parametric study. The evolution of the constraint τu according to anchoring (t) is represented to the figure (8) for the three lengths.

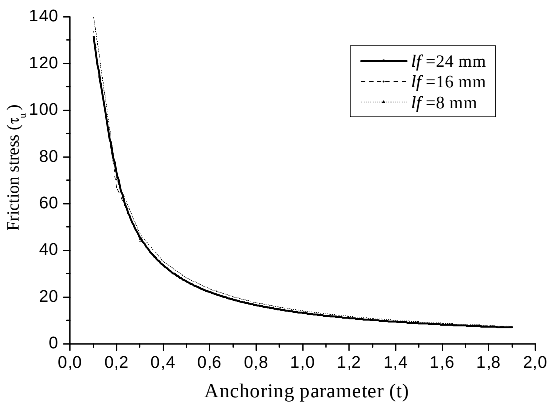


Fig. 8. The evolution of τu according to theparameter of anchorage t.

The value of τu adopted corresponds to the lower limit of the curve (with the stabilization of this one). When the value of T becomes higher than 1.4 and until to 2, the constraint of adhesion τu does not evolve almost any more. The three curves converge towards the same value (τu = 7.92 MPa).

This last value is then introduced into the relations stress-strains (23, 24) suggested in [1, 2]. These relations are then modified for better taking into account the undulating fibers. From the expressions of opening of the cracks, the authors then made a passage to stress-strains relations. These relations are given by (23).

$$\begin{cases} \sigma = E_{ct}\,\varepsilon & si \quad 0 \le \varepsilon \le \varepsilon_{ft} \\ \sigma = \sigma_{uc} - [\sigma_{uc} - f_{ft}]\,\dfrac{(\varepsilon - \varepsilon_u)^6}{(\varepsilon_{ft} - \varepsilon_u)^6} & \\ & si \quad \varepsilon_{ft} \le \varepsilon \le \varepsilon_u \\ \sigma = \sigma_{uc}\,[1 - \dfrac{(\varepsilon - \varepsilon_u)^6}{(\varepsilon_r - \varepsilon_u)^6}] & si \quad \varepsilon_u \le \varepsilon \le \varepsilon_r \end{cases} \qquad (23)$$

$$\begin{cases} \sigma = E_{ct}\,\varepsilon & si \quad 0 \le \varepsilon \le \varepsilon_{ft} \\ \sigma = \sigma_{uc} - [\sigma_{uc} - f_{ft}]\,\dfrac{(\varepsilon - \varepsilon_u)^6}{(\varepsilon_{ft} - \varepsilon_u)^6} & \\ & si \quad \varepsilon_{ft} \le \varepsilon \le \varepsilon_u \\ \sigma = a\,\sigma_{uc}\,[1 - \dfrac{(\varepsilon - \varepsilon_u)^b}{(\varepsilon_r - \varepsilon_u)^b}] & si \quad \varepsilon_u \le \varepsilon \le \varepsilon_r \end{cases} \qquad (24)$$

The effort of pulling and the shear stress are given respectively by the equations (21,22). According to these considerations the equations (23) are transformed into equations (24). They thus make it possible to take account of the corrugated geometrical shape of fiber. [3]

With; $\alpha = 0.8$, $\beta = 7$ and $s_{uc} = w\, q_0\, \mathbf{l}_f\, \dfrac{t_u}{f}$

The parameters α and β are given starting from the confrontation of the results (24) to experimental results. The model suggested is introduced into nonlinear computation software, until rupture, of a section of beam [CMP]. [6]

4.2 Confrontation of the relation stress-strain in direct pulling

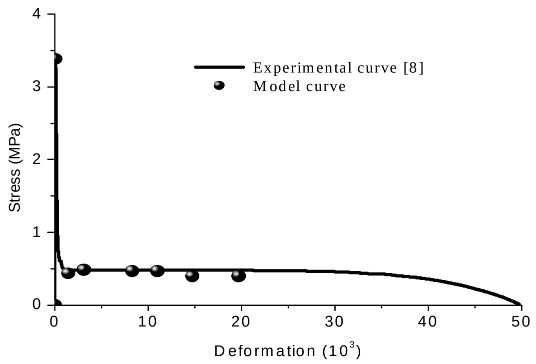
The comparison of these relations is carried out compared to the results obtained on cylindrical test-tubes (length 320 mm, diameter 160 mm) tested in direct pulling by Zhan [8]. The deformations of the test-tubes tested were obtained, by Zhan, by dividing the opening of the crack measured by the length of the base of measurement of the extensometers fixed on the test-tube (140 mm). The test-tubes are out of concrete reinforced with corrugated metal fibers. The percentage in volume of fibers is 0, 31%. The mechanical characteristics of the concrete and fibers are given to the tables (1 and 2). The comparison of the results is illustrated with the figure (09) and detailed appears about it (10). After the linear elastic phase, there is a fall of constraint which is stabilized in the form of a residual resistance until approximately 3% of the deformation. The behavior is approximate in a satisfactory way.

Tab. 1 Mechanical characteristics of the composite (Concrete reinforced with steel fibers).

	F_{ci}(MPa)	F_{ti} (MPa)	E_{bo} (GPa)	R_b	R_c	ε_{rt} ‰	ε_o ‰	ε_{cu} ‰
BFON	47,7	2,94	38,18	1,6	0,7	- 50	2,1	3,5

Tab. 2 Characteristics of the fibers.

	E_f (GPa)	l_f (mm)	ω(%)	ϕ (mm)	ε_u ‰	τ_u (MPa)
BFON	200	60	0,31	1	- 0,74	7,92



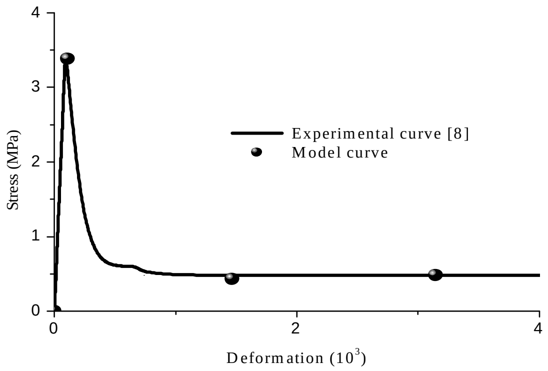


Fig. 09 Comparison with the results obtained by [8] (Fiber length lf = 60 mm) **Fig. 10** Detail of figure 11

5 Conclusions

The incorporation of steel fibers in a matrix concrete gets for the composite a ductile behavior in traction. By effect of sewing of the cracks, the fibers limit their spread and absorb a certain quantity of energy. This phenomenon a function of the geometry and the mechanical characteristics of fiber used. In order to determine the contribution of fibers well to the behavior of the composite in traction, it is essential to take account of the shape of fibers (right, with or without anchoring at the ends or corrugated). Within the framework of this work, one was interested in corrugated fiber. Modeling is based on the effect of rubbing and friction and takes into account the undulations of fiber. A parametric study made it possible to model the friction of fiber undulated in a concrete matrix and to highlight the evolution of the friction stress (τ_u) according to the parameter of anchoring (t). This modeling is introduced by adaptation of the relations stress-strains suggested for a concrete reinforced with fibers in traction by [1], [2]. The software, making it possible to follow the nonlinear behavior until the rupture of a concrete section, proposed by [7], is used to validate this modeling. The results of confrontations carried out are satisfactory.

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