MODELLING QUASI-BRITTLE FAILURES OF MATERIALS

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Abstract:

Quasi-brittle materials (eg concrete and some ceramics) distinguish themselves by a dissipation before to cracking being with no or irrelevant permanent deformations. Atomic decohesions are the leading mechanisms to cause a rapid growth of a crack by instability. Anyhow, the quasi-brittle materials may possess some reversible slidings of nano- or microcracks taken into consideration as initial defects to induce a loss of energy by friction. Both mechanisms are unsafe because there is no precursor to wake watchers up.

Keywords: crack propagation, decohesion, failure, structure response, two-scale model

1 Introduction

Approximately 1920, A.A. Griffith introduced the intention of energy of decohesion applied in fracture mechanics by dint of the strain energy release rate variable. In 1939, W. Weibull brought in the concept of statistical distribution of initial defects originating a probability simulation of failure expressed in terms of stress and of the volume taken in account. From that time, many models of quasi-brittle materials are grounded on initial microdefects or are damage simulations. To predict the crack initiation in construction subject to intense loadings, for last decades, the promising method-continuum damage mechanics has been worked out.

For brittle materials, the characteristic subjects are the general fracture criteria, debonding, probabilistic approaches, delamination of composites, and dynamic failures.

2 Practical factors

The strain of brittle and quasi-brittle materials to failure is small ($\varepsilon_R < 2 \cdot 10^{-2}$) and their toughness is at best on the order of several MPa√m. The design of construction prepared of brittle materials is arduous for is no space for plastic shakedown in the instance of overloading but reinforcements assist.

For rough estimations, mesostress criteria may be applied, but for precise estimations the statistical distribution of internal defects must be taken into consideration. Unfortunately, they cannot be precisely evaluated by nondestructive methods. A possible alternative is to deduce a probabilistic information from the scatter of test results by an inverse method for high cycle fatigue (F. Hild 1994). To illustrate this point, let us determine the probability density of the relative size of defects in a brittle material on which many failure tests have been performed.

The following simplified assumptions are made:

- 10 to 20 failure tests are available in simple tension on the same geometry.
- For each specimen, the area density of the initial defects in the plane normal to the stress where the failure will occur is D_0. Then the failure stress σ_R is simply given by the effective stress concept. For brittle failures, σ_R is related to an initial damage,

$$\sigma_R = \sigma_u (1 - D_0) \quad \text{or} \quad D_0 = 1 - \frac{\sigma_R}{\sigma_u} \tag{1}$$

where σ_u is the rupture stress of the material without any defect. On a practical level, it is the maximum value of σ_R measured, assuming that in the set of specimens, at least one has no defects (or only some very small ones).

The damage D_0 is now a random variable (as is σ_R) for which the probability density resulting from the tests is P(σ_R). The probability for σ_R to have values bounded by σ_a and σ_b is

$$P(\sigma_a < \sigma_R < \sigma_b) = \int_{\sigma_a}^{\sigma_b} P(\sigma_R)\, d\sigma_R \tag{2}$$

It is also the probability for the decreasing function $D_0(\sigma_R)$=1-σ_u/σ_R to have values bounded by $D_0(\sigma_b)$= D_{0b} and $D_0(\sigma_a)$= D_{0a}. Considering the inverse function $\sigma_R = \sigma_u(1 - D_0)$, we then have

$$P(D_{0b} < D_0(\sigma_R) < D_{0a}) = \int_{\sigma_u(1-D_{0b})}^{\sigma_u(1-D_{0a})} P(\sigma_R)\, d\sigma_R \tag{3}$$

or by the change of variable σ_R= σ_u(1- D_0), we then have

$$P(D_{0b} < D_0(\sigma_R) < D_{0a}) = -\int_{D_{0a}}^{D_{0b}} P(\sigma_u(1 - D_0)) \cdot (-\sigma_u dD_0) \tag{4}$$

which shows that the probability density of initial damage D_0 is

$$P(D_0) = \sigma_u P(\sigma_u(1 - D_0)) \tag{5}$$

For example, if σ_R obeys a Gaussian distribution,

$$P(\sigma_R) = \frac{1}{\bar{\bar{\sigma}}_R \sqrt{2\pi}} \exp - \frac{(\sigma_R - \bar{\sigma}_R)^2}{2\bar{\bar{\sigma}}_R^2} \tag{6}$$

where $\bar{\sigma}_R$ is the mean value of the failure stress and $\bar{\bar{\sigma}}_R$ its standard deviation, the probability density of D_0 is given by

$$P(D_0) = \frac{\sigma_u}{\bar{\bar{\sigma}}_R \sqrt{2\pi}} \exp - \frac{(\sigma_u(1 - D_0) - \bar{\sigma}_R)^2}{2\bar{\bar{\sigma}}_R^2} \tag{7}$$

or

$$P(D_0) = \frac{s_u}{\frac{\bar{\bar{s}}_R}{s_u}\sqrt{2p}} \exp - \frac{\left(D_0 - 1 + \frac{\bar{s}_R}{s_u}\right)^2}{2\left(\frac{\bar{\bar{s}}_R}{s_u}\right)^2} \qquad (8)$$

also a Gaussian distribution for D_0. The mean value of D_0 is:

$$\bar{D}_0 = 1 - \frac{\bar{s}_R}{s_u} \qquad (9)$$

and its standard deviation is

$$\bar{\bar{D}}_0 = \frac{\bar{\bar{s}}_R}{s_u} \qquad (10)$$

There is a simple manner to get an input for stochastic failure analysis. Instead of employing a continuous probability rule (such as a Gaussian law), a discrete numerical interpretation is possible, being demonstrated in Fig. 1 where the histogram of initial damage is inferred in a simple fashion from a set of stresses to fracture by $D_{0i}=1-\sigma_{Ri}/\sigma_u$. If i_{max} is the number of tests (for the most part low, here i_{max} = 20), applying $p_{max} \approx 2+\sqrt{i_{max}}$ (here p_{max} = 6) as the number of intervals for the histograms is a satisfactory compromise between excessively low (shallow distribution) and exceedingly large numbers of values (also shallow distribution).

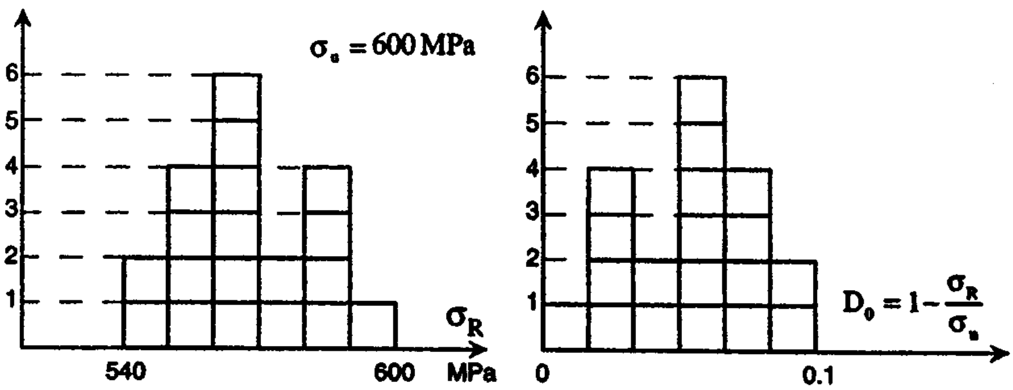

Fig. 1 Histogram of initial damage D_0 deduced from the histogram of failure stress

3 Homogenized characteristics of quasi-brittle substances

Collapse of brittle or quasi-brittle constituents is principally determined by initial defects, which are random in their size and space classification. Accordingly, the exactitude of prediction is frequently inadequate for two reasons: high dispersion of fundamental test issues to recognize the parameters and ignorance of the state of initial defects. However, discussions of probabilities give a sense to these unsteadinesses.

For brittle materials or interfaces:

- In absence of any information other than an ultimate stress, use the damage equivalent stress criterion (and not the von Mises stress),

$$s^{*} = s_{eq} R_v^{1/2} = s_u \quad \text{with} \quad R_v = \frac{2}{3}(1+v) + 3(1-2v)\left(\frac{s_H}{s_{eq}}\right)^{1/2} \tag{11}$$

  Introducing the microdefects closure is an improvement if some compression occurs.

- The Mazars damage equivalent strain is better for concrete:

$$\in^{*} = \sqrt{\langle \in \rangle^{+} : \langle \in \rangle^{+}} = \frac{s_u^{+}}{E} = v\sqrt{2}\,\frac{s_u^{-}}{E} \tag{12}$$

- For interfaces a debonding criterion which needs two material parameters may be applied:

$$\left\langle s_{33}\left|s_{33}\right| + \left(\frac{s_R^I}{t_R^I}\right)^2 \left(s_{13}^2 + s_{23}^2\right)\right\rangle^{1/2} = 1 \tag{13}$$

- The Weibull model should be used to characterize the probability of failure but the material parameters need 10 to 20 specimens for their identification:

$$P_F = 1 - \exp\left\{ -\frac{V_{eff}}{V_0} \left[\frac{s^{*}}{s_w}\right]^{m_w} \right\} \tag{14}$$

Using the two-scale damage model is a way to predict the rupture of quasi-brittle materials that occurs when a dissipation prior to cracking exists (if some fatigue results complete the identification database) but several other damage models have their specific applications:

- The Mario model with or without microdefects closure effects is a general model valid for most materials while the anisotropic damage model is suitable for concrete
- The Laborderie model with permanent strains and microdefects closure effects in dynamics (seismic effects on civil engineering structures)
- Mesomodels for composites where three damage variables are considered
- Probabilistic models for ceramics and fragmentation in dynamics

Finally, elasticity and damage models for reinforced concrete are of main importance in civil engineering. Due to the size of the structures, it is interesting in FE computations to avoid meshing the steel bars and the concrete body separately. Homogenization procedures give the equivalent elastic properties of heterogeneous materials. They apply to undamaged reinforced concrete and uniaxial bar reinforcement, yielding orthotropic elasticity characteristics. For example, if the steel spacing is the same in the two transverse directions, transverse isotropy is obtained with longitudinal and transverse Young's moduli E_L and E_T, Poisson ratios ν_{LT} and ν_T shear moduli G_{LT} and G_{TT}:

$$\begin{aligned}
E_L &= E_L\left(f, E_c, E_s, ...\right)\\
E_T &= E_T\left(f, E_c, E_s, ...\right)\\
n_{LT} &= n_{LT}\left(f, E_c, E_s, n_c, n_s, ...\right)\\
n_T &= n_T\left(f, E_c, E_s, n_c, n_s, ...\right)\\
G_{LT} &= G_{LT}\left(f, E_c, E_s, n_c, n_s, ...\right)\\
G_{TT} &= \frac{E_T}{2\left(1+n_T\right)}
\end{aligned} \tag{15}$$

whose specific expressions depend on the homogenization procedure. They are functions of steel volume fraction (f), the elastic properties of concrete (E_c, ν_c) and steel (E_s, ν_s).

The question of coupling with damage arises then and one needs to take into account the microcrack closure effect in concrete. One of the simplest possible models uses the anisotropic and damage framework, but extended to transverse anisotropy,

$$\begin{aligned}
r y^* = {} & \frac{\langle s_{11}\rangle^2}{2\tilde{E}_L^+} + \frac{\langle -s_{11}\rangle^2}{2\tilde{E}_L^-} + \frac{\langle s_{22}\rangle^2 + \langle s_{33}\rangle^2}{2\tilde{E}_T^+} + \frac{\langle -s_{22}\rangle^2 + \langle -s_{33}\rangle^2}{2\tilde{E}_T^-}\\
& - \frac{n_{LT}}{E_L}\left(s_{11}s_{22} + s_{11}s_{33}\right) - \frac{n_{TT}}{E_L} s_{22}s_{33} + \frac{s_{12}^2 + s_{13}^2}{2G_{LT}} + \frac{s_{23}^2}{2G_{TT}}
\end{aligned} \tag{16}$$

where the coupling with damage is reduced to the minimum by making the damage variable D_L act on the longitudinal modulus, the damage variable D_T on the transverse modulus, and by neglecting the coupling of the ratios ν_{ij}/E_j and the shear moduli with damage:

$$\begin{aligned}
\tilde{E}_L^+ &= E_L\left(f, E_c\left(1-D_L\right), E_s, ...\right)\\
\tilde{E}_L^- &= E_L\left(f, E_c\left(1-hD_L\right), E_s, ...\right)\\
\tilde{E}_T^+ &= E_T\left(f, E_c\left(1-D_T\right), E_s, ...\right)\\
\tilde{E}_T^- &= E_T\left(f, E_c\left(1-hD_T\right), E_s, ...\right)
\end{aligned} \tag{17}$$

where **h** is the microdefects closure parameter.

The damage evolution laws are written as $D = D(Y)$ laws

$$D_L = D_L\left(Y_{L\,\max}\right) \text{ and } D_T = D_T\left(Y_{T\,\max}\right) \tag{18}$$

with $Y_{L\ max}$ and $Y_{T\ max}$ as the maximum values reached during the loading of the longitudinal and transverse strain energy release rates

$$Y_L = r\,\frac{\partial y^*}{\partial D_L} \quad \text{and} \quad Y_T = r\,\frac{\partial y^*}{\partial D_T} \tag{19}$$

Applying the simple mixture laws

$$E_L = f\,E_s(1-f)E_c \quad \text{and} \quad E_T = \left(\frac{f}{E_s} + \frac{1-f}{E_c}\right)^{-1} \tag{20}$$

yields

$$Y_L = \frac{(1-f)E_c}{2}\left[\frac{\langle s_{11}\rangle^2}{\tilde{E}_L^{+2}} + h\frac{\langle -s_{11}\rangle^2}{\tilde{E}_L^{-2}}\right] \tag{21}$$

$$Y_T \approx \frac{1-f}{2E_c}\left[\frac{\langle s_{22}\rangle^2 + \langle s_{33}\rangle^2}{(1-D_T)^2} + h\frac{\langle -s_{22}\rangle^2 + \langle -s_{33}\rangle^2}{(1-hD_T)^2}\right]$$

that take into consideration for concrete, (by means of h), the much lower growth in compression than in tension.

4 Conclusion

The basic aspects of continuum damage mechanics given are concentrated on the application of damage models destined for quasi-brittle materials, namely, in particular, on the numerical analysis of fracture: (i) anisotropic damage model for concrete, (ii) failure of pre-stressed concrete structures, (iii) seismic response of reinforced concrete construction, and (iv) failure of ceramic matrix composites (damage and delamination).

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