

DETERMINATION OF TENSILE PART OF FIBRE CONCRETE STRESS-STRAIN DIAGRAM FROM BENDING TEST MEASUREMENTS

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Abstract:

The paper describes the recommended tests of fibre concrete elements for determination of tensile characteristics. The results of the standardized tests with test beams of recommended dimensions can be used for determination of the tensile part of the characteristic stress-strain diagram of fibre concrete. Examples of concrete with steel and synthetic fibres are presented.

Keywords: Fibre concrete (FC), testing, stress-strain diagram, cracking, tensile strength

1 Introduction

Fibre concrete is a composite building material. Its matrix consists of plain concrete reinforced with fibres, which makes the composite stiffer. The fibres can be made of various materials with various shape and size. The amount of the fibres in the fibre concrete should be set so that it yields a homogeneous material. This helps to improve some of the mechanical properties of plain concrete. The volume ratio is usually given in percents and it defines the volumetric ratio of fibres in fibre concrete.

The type of the fibres used affects the mechanical parameters of fibre concrete. There is a large selection of fibres produced and this it is necessary to verify experimentally its effect on the decisive properties of fibre concrete and to determine the characteristic values of the parameters necessary for design of fibre concrete structures.

The significant differences in behaviour of fibre concrete elements are demonstrated after macro-cracks occur in the tensile regions. The tensile forces in this region are transferred by the dispersed fibres after the macro-crack occurs. The effect of the fibres can be accounted for by considering the fictitious stress in the tensile regions, which represents the residual tensile stress, or the uniformly distributed stress which is equal to the tensile strength of the fibre concrete. The quasi-linear elastic behaviour of fibre concrete structural element changes at the macro-crack initiation into the quasi-plastic behaviour. This change in behaviour needs to be considered in the analysis of fibre concrete structures.

2 Recommended tests of fibre concrete elements

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Fibre concrete is a material with increased requirements on production technology. The fibres affect the fibre concrete properties, primarily the strength characteristics in tension and ductility. The fundamental requirement for application of fibre concrete in structures is the uniform distribution of fibre in the composite.

In order to verify homogeneity of the composite, a specimen of recommended shape and size should be tested by a suitable method. Such a method is represented by the four-point bending test when a beam specimen of the dimensions 150x150x700 mm is used, see Fig. 1.



Fig. 1 Standard bending test setup (recommended beam specimen, cross section 150×150 mm, length 700 mm, span L = 600mm)

The reasons for selection of this testing method are:

- The recommended size of the specimen is sufficient of testing of fibre concrete with any of the currently available types of fibres (the length is important).
- The section of the beam specimen which is subjected to the maximum bending moment is sufficient (the length between the two points of action (*a*) is 200 mm).
- High ratio between the theoretical span and the height of the specimen (h). Flexural slenderness of the beam specimen is 600/15 = 4, which is more similar to the beam elements than when shorter specimens are used.
- The specimen does not fail due to the combination of shear and bending, since a/h = 200/15 = 4/3.

The beam specimen is proposed as the standard specimen for the standard test of fibre concrete due to the advantages listed above. The loading is controlled by deformation during testing.

Based on the results of the described testing method, the basic mechanical properties in tension of the tested fibre concrete can be derived according to [1], or [2]. At least three, or better six, specimens are tested, and the result is obtained in the form of the loaddeflection diagram (also know as the diagram of resistance of standard beam specimens). The mean diagram is determined with the mean value of the relation $(F_R - d_t)_m$ and the characteristic resistance diagram with the characteristic relation $(F_R - d_t)_k$ as the fifth percentile of the measured results. Figs. 2 a) b) and c) show the examples of results



LOAD F_R [kN] 60 a) (CLS), Mean value 50 Characteristic value F_R 40 F_{Rk} F_{Rk,res,3} (CLS)_k F_{Rm,i} 30 20 F_{Rk,res,1} 10 K_{m} (minF_{Rk,res,1}) 0 0,0 $\delta_{tk,c} \ \delta_{tm,c}$ 1,0 δ_{t3} 2,0 3,0 $\delta_{t1}=3,5mm$ 4,0 5,0 DEFLECTION δ_t [mm] 30 (CLS)_m b) Mean value F_{Rm} (CLS)_k Charakteristic value F_{Rk,c} 20 LOAD F_R [kN] F_{Rm} F_{Rk,res,2} F_{Rk,res,3} 10 F_{Rk,res,1} K_m (minF_{Rk,res,1}) 0 $\delta_{tk,c} \ \delta_{tm,c} \ \delta_{t2}$ $\delta_{t1} = 3,5mm$ $\boldsymbol{\delta}_{t3}$ 0,0 4,0 5 DEFLECTION δ_t [mm] 1,0 2,0 3,0 5,0 50 (CLS)_m Mean value c) F_{Rm,c} (CLS)_k Characteristic value 40 F_{Rk.} LOAD F_R [kN] 30 F_{Rm,res,1} 20 10 F_{Rk,res,1} K_m (minF_{Rk,res,1}) F_{Rk,res,3} F_{Rk,res,2} 0 δ_{tm} δ_{t1} =3,5mm 0,0 $\delta_{\!t2}$ δ_{t3} 2,0 1,0 3,0 4.0 5,0 DEFLECTION δ_t [mm]

obtained with specimens of different fibre concrete in terms of the mean and characteristic resistance diagrams. The crack limit state is indicated by CLS in the diagrams.

Fig. 2 Mean and characteristic resistance diagrams of specimens with different types of fibre concrete:



a) FC 70/77, steel fibres Dramix RC80/30 BP $r_{V,f} = 1.0 \%$ b) FC 40/75, steel fibres Arcelor HE 1/50 $r_{V,f} = 0.51 \%$ c) FC 60/65, synthetic fibres Forta Ferro $r_{V,f} = 1.0 \%$

The shapes of the diagrams shown in Fig. 2 are different. This is caused mainly by the volume ratio of fibres $r_{V,f}$, type and material of the fibre, but also by the quality of the basic concrete matrix, which is denoted by the strength class and the homogeneity of the fibre concrete microstructure, which affects the deviation of the measured data.

3 Evaluation of characteristic resistance diagrams

The characteristic resistance diagrams can be used for assessment of the behaviour of fibre concrete in the tensile region of the structural element and also for explanation of the effects of the mechanical properties of fibre concrete. The characteristic stress-strain diagrams of fibre concrete in tension $(\mathbf{s}_{fc,t} - \mathbf{e}_{fc,t})_k$ can be derived from the characteristic resistance diagram under the following conditions:

- Quasi-linear elastic behaviour (QLE) can be assumed until the macro-crack initiation (CLS).
- Quasi-plastic behaviour (QPL) is assumed from the macro-crack initiation till the instant when the maximum deflection at the mid-span $d_{t,1}$ is reached (see Fig. 2 where $d_{t,1} = 3.5$ mm).

The differences between the mean and characteristic values of the resistance diagrams $(F_{\rm Rm} - F_{\rm Rk})$ at the same deflection $d_{\rm t}$ define the deviations in the measured data. Based on these differences, especially after the macro-crack initiation, the following effects can be recognized:

- Material and type of fibres, volume ratio of fibres in the concrete matrix $r_{V,f}$, which increase the tensile resistance.
- Compressive strength classes of fibre concrete, which affect bond between the fibre and the matrix, or the mechanical anchorage of the fibre in the matrix using the designed ends.
- Correct mix proportions in the concrete matrix in order to ensure homogeneous distribution of the fibre and ensuring acceptable deviations in the measured data for derivation of the characteristic resistance diagrams of fibre concrete.

Fig. 2 shows how the above effects change the characteristic values of resistance $F_{\text{Rk},}$ res after the macro-crack initiation, or the entire shape of the resistance diagram $(F_{\text{R}} - d_{\text{t}})_{\text{k}}$. The local maximum of resistance $F_{\text{Rk}, \text{ res},3}$ at the deflection $d_{\text{t},3} < d_{\text{t},1}$ can also occur, when the value of the deflection $d_{\text{t},3}$ in the characteristic diagram is different from that in the mean resistance diagram.

Some more series of specimens with different type of fibre concrete were tested. Evaluated diagrams ($F_{\rm R}$ - $d_{\rm t}$) of tests are published at [3].



4 Derivation of characteristic stress-strain diagram of fibre concrete in tension

For derivation of the tensile part of the characteristic stress-strain diagram of fibre concrete it is necessary accept the hypotheses about the stress distribution in the critical cross section of the fibre concrete element at each loading stage.

Until the macro-crack initiation, when the tensile region of fibre concrete element show quasi-linear elastic behaviour, the linear elastic calculation can be used. The stress distribution shown in Fig. 3 a) can be assumed until the tensile strength of fibre concrete $f_{fc,t,fl}$ is reached at a surface of the fibre concrete element.



Fig. 3 Stress distribution in beam specimen: a) uncracked critical section b) cracked critical section

The correct stress distribution in the critical section after macro-crack initiation cannot be express easily. The location of the neutral axis changes and thus the stress distribution in the cross section also changes. The concrete composite rapidly loses its resistance in tension due to the propagating crack in the tensile region, and only at the very near vicinity of the neutral axis the concrete is not ruptured, which is given by $S_{fc,t} \leq f_{fc,t,fl}$. The crack is then bridges only by the fibres which transfer the tensile forces across the expanding crack width. The sum of the tensile force in the fibres can be expressed by the idealized equivalent uniformly distributed stress $S_{fc,t,i} = f_{fc,t,eq,i}$ over the entire tensile region, which is the equivalent strength of fibre concrete in uniaxial tension, see Fig. 3 b).

The value of this strength depends on the value of the strain $e_{fc,t,i}$ distributed along the pulled edge of the cross section at the location of quasi-plastic hinge, which occurs as the result of expansion of the macro-crack. The value $e_{fc,t,i}$ can be derived from the deflection at the mid-span $d_{t,i}$ for the resistance $F_{Rk, res,i}$ under the assumption that the cross section remains planar even after the rotation of the quasi-plastic hinge with zero strain at the neutral axis. The neutral axis is at the distance $x_{r,i}$ from the compressed edge of the critical cross section, which is considered in the mid-span of the beam element.

Under the following assumptions, simplified derivation of the stress distribution in the tensile region of the stress-strain diagram from the characteristic resistance diagram $(F_{\rm R} - d_{\rm t})_{\rm k}$ can be made:

• Until the macro-crack initiation, it is a linear relation $(s_{fc,t} - e_{fc,t})_k$ up to the flexural tensile strength of fibre concrete in tension $(s_{fc,t} = f_{fc,tk,fl})$ at the derived strain at the pulled edge.



2)

- After the macro-crack initiation, the decisive parameter can be:
 - either the maximum strain $e_{fc,t,1}$ derived for the agreed value of deflection of the beam specimen (e.g. for $d_{t1} = 3.5$ mm) and constant residual stress can be assumed over the entire tensile region, the strength equal to the equivalent strength of fibre concrete under uniaxial tension according to equation (2),

$$f_{\rm fc,tk,res,1} = f_{\rm fc,tk,eq,1} \tag{6}$$

- or to derive the corresponding characteristic value $f_{\text{fc,tk,eq,3}}$ and the strain $e_{\text{fc,t,3}}$ at deflection d_{t3} for the local maximum characteristic residual resistance $F_{\text{Rk, res,3}}$ (see Fig. 2). The derived equivalent strength of fibre concrete under uniaxial tension $f_{\text{fc,tk,eq,3}}$. can be assumed over the entire tensile region in the critical cross section given by $(h - x_r)$.

Fig. 4 shows the characteristic stress-strain diagrams of fibre concrete in tension derived from the characteristic resistance diagrams shown in Fig. 2.

The stress-strain diagrams show the principal differences in the tensile parts of the characteristic stress-strain diagrams derived from the resistance diagrams shown in Fig. 2. Especially obvious is the influence of material of fibres on the quasi-linear elastic behaviour of fibre concrete before the macro-crack initiation (comparison of Figs. 4a and 4c) and the effect of loading rate. In the tests shown in Figs. 2b and 4b, the loading rate was reduced to 1/10 after the crack limit state was violated, as compared to the constant loading rate shown in Figs. 2a and 4a.

The effect of material of the fibres on the differences in the quasi-plastic behaviour after the crack initiation is apparent when Figs. 4a and 4b are compared with Fig. 4c.

The volume rate of fibres and the compressive strength class of fibre concrete also affect the equivalent tensile strength. Fibre concrete with high strength classes with steel and synthetic fibres with volume rate of $r_{V,f} = 1.0$ % show similar values of tensile strength at the macro-crack initiation. However, after the macro-crack initiation the fibre concrete with synthetic fibres shown lower equivalent tensile strength when compared with steel fibre concrete with the volume ratio of steel fibre of $r_{V,f} = 0.51\%$.

5 Conclusions

The tensile part of the characteristic stress-strain diagram derived by the described procedure can be used for design of plain fibre concrete structures. The compressed part of the stress-strain diagram of fibre concrete is obtained similarly to the ordinary concrete. For fibre concrete structures reinforced with traditional rebars and for prestressed fibre concrete structures it is necessary to use the characteristic stress-strain diagram along with the characteristic stress-strain diagram of the used reinforcing bars for accounting for the bond.

For simplification of the calculation of the bond, the tensile part of the stress-strain diagram can be divided. Quasi-linear elastic behaviour can be assumed before the macro-crack occurrence and quasi-plastic behaviour can be assumed after the macro-crack initiation with the simplification given in Fig. 3b.





Fig. 4 Tensile part of characteristic stress-strain diagram of fibre concrete - see Fig.2a,b,c (solid= simplified distribution, dashed = multi-linear distribution for more precise derivation diagram)

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