DESIGN OF REINFORCEMENT IN STEEL FIBRE CONCRETE WITH RESPECT TO CRACKS

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Abstract

When steel bars are placed in a concrete structure, the evaluation of cracks and crack spacing is generally required in the serviceability limit state. According to more or less aggressive conditions, crack width shall be limited in order to avoid, for instance, corrosion of reinforcement. The presence of steel fibres in the concrete cast may help to achieve this goal since fibres remarkably increase the bridging across the crack. In the article the design of the reinforcement in steel fibre concrete with respect to required width of crack using the modified procedures described in EN 1992-1-1[1] and in DIN 1045-1 [2] is described. For this design of the reinforcement a computer program has been developed, described also in the article.

Keywords: width of crack, design of reinforcement, steel fibre reinforcement, concrete.,

Introduction

Concrete has high compressive strength and good durability, but low tension strength and therefore it tends to crack under tensile loading and results in brittle failure. Internal fibre reinforcement can improve the tensile strength and ductility of concrete beyond the elastic limit. Fibre reinforced concrete can be categorized as a material with softening and hardening in direct tensile loading.

While in the plain concrete bending element one crack propagates and leads to brittle failure, for fibre concrete it may be assumed that the critical section is capable to resist tensile strain and usually can form other cracks.

Fibres in concrete could prevent cracking or the limit crack width for service loads. Cracking is connected with other features in the serviceability limit state – smaller deflection and higher durability. Due to usually higher tensile strength of fibre concrete cracks occur later and flexural stiffness which is influenced by cracking also drops later. Durability depends on permeability, which is affected by cracking and porosity of concrete.

Fibre concrete can be considered as a composite material with specific material properties which can be predicted according to the class of concrete and the type and amount of fibres used in the mixture. The behaviour of fibre concrete can be prescribed by

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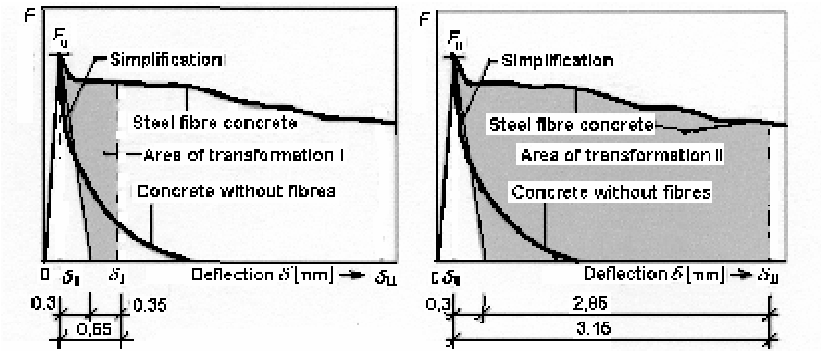
relationship between the force *F* and the deflection *d* by the three or four points bending (see e.g. Fig. 1a and b).

The connection of ordinary reinforcement and steel fibres is very useful, as steel fibres reduce the amount of reinforcement that is necessary for securing the required width of cracks. The required width of cracks depends usually on the exposure class related to environmental conditions in which the structure is located. The required width of cracks related to the exposure class is given in standards, but for thin steel fibres it is recommended to choose the values reduced about 0,1 mm. The exposure class requires a prescribed class and composition of concrete as required in standards.



a) without cracks – state I, b) with cracks – state II

Fig. 1 Relationship between the force *F* and the deflection *d* of bending element

Procedure according to EN 1992-1-1

According to EN 1992-1-1 [1] two types of cracks namely in dependence to differences by cracking are distinguished:

- cracks due to restricted intrinsic deformations of the element (for instance from heat of hydratation, shrinkage, change of temperature, etc.); it is the mode of cracks influenced by restricted strain – early cracks;

- cracks predominantly due to loading; it is the mode of cracks influenced by loading - later cracks.

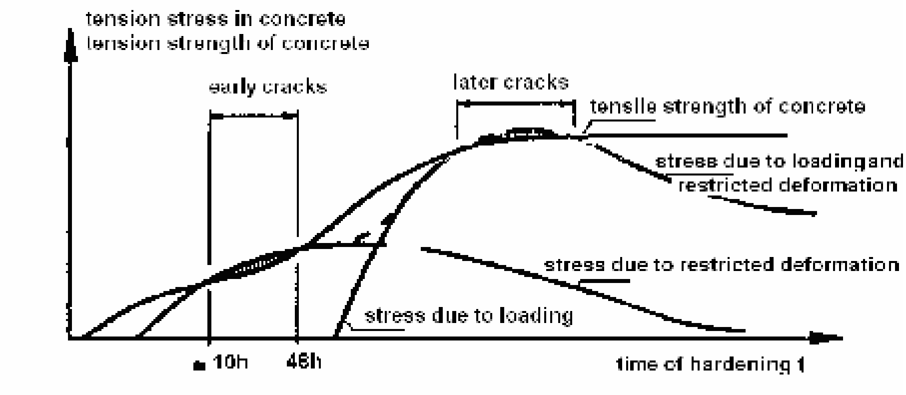


Fig. 2 Time dependent rise of tension stress due to restricted deformation and loading as compared to the increase of tension strength in concrete member [3]

As far as the limitation of width of cracks is required, in areas where tension is expected, it is necessary place the reinforcement for this required limitation. The amount of reinforcement it is possible to determine from the equilibrium of tension force in concrete $F_{cr} = A_{ct}\, f_{ct}$ (where f_{ct} is tension strength of concrete closely before the creation of crack) and tension force in reinforcement $F_s = A_s\, s_s$ (where s_s is the stress in steel which satisfy the required width of crack). It is possible to write the condition of equilibrium

$$A_{ct}\, f_{ct} = A_s\, \sigma_s \tag{1}$$

Taking into consideration that by majority of concrete members the cracks arise due to restricted intrinsic deformations, the standard EN 1992-1-1[1] require the minimum reinforcement $A_{s,min}$ which in the case of restricted intrinsic deformation by escape heat of hydratation is calculated from the tension forces by formation of the first (early) crack.

Unless a more rigorous calculation shows lesser areas to be adequate, the required minimum areas of reinforcement $A_{s,min}$ may be calculated in fibre concrete using the formula in [1], but modified in

$$A_{s,min}\, \sigma_s = F_{cr} \tag{2}$$

where

$$F_{cr} = (k_c\, k\, f_{ct,eff} - f_{ctk,eq,t,I})\, A_{ct} \tag{3}$$

$A_{s,min}$ is the minimum area of reinforcing steel within the tensile zone;

A_{ct} the area of concrete within tensile zone; the tensile zone is that part of the section which is calculated to be in tension just before formation of the first crack;

s_s the absolute value of the maximum stress permitted in the reinforcement immediately after formation of the crack which satisfy the required crack width;

the mean value of the tensile strength of the concrete effective at the time when the cracks may be expected first to occur: $f_{ct,eff} = f_{ctm}$ or lower; if cracking is expected earlier than 28 days $f_{ct,eff} = f_{ctm}(t)$; according to [2] it is possible for early cracks consider $f_{ct,eff} \cong 0{,}5\, f_{ctm}$;

$f_{ctk,eff,t,I}$ the characteristic value of equivalent tensile strength of fibre concrete for the area transformation I, in the time t;

k the coefficient which allows for the effect of non-uniform self-equilibrating stresses, which lead to a reduction of restraint forces:
k =1,0 for webs with $h \leq 300$ mm or flanges with widths less than 300 mm,
k=0,65 for webs with $h \geq 800$ mm or flanges with widths greater than 800 mm,
intermediate values may be interpolated;

k_c koefficient which takes account of the stress distribution within the section immediately prior to cracking and of the change of the lever arm:
for pure tension $k_c = 1{,}0$,
for bending $k_c = 0{,}4$,
for bending combined with axial forces see [1];

Width of crack depends not only on the stress s_s in the steel reinforcement and on strain belongs to this stress, but also on the other parameters, as follow from the next.

The crack width w_k may be calculated according to [1] from Expression

$$w_k = s_{r,max} (e_{sm} - e_{cm}) \tag{4}$$

where

$s_{r,max}$ is the maximum crack spacing;

e_{sm} the mean strain in the reinforcement under the relevant combination of loads, including the effect of imposed deformations and taking into account the effects of tension stiffening;

e_{cm} is the mean strain in the concrete between cracks

The difference of mean strains ($e_{sm} - e_{cm}$) may be calculated from the expression

$$\varepsilon_{sm} - \varepsilon_{cm} = \frac{\sigma_s - k_t \dfrac{f_{ct,eff}}{\rho_{eff}} \left(1 + \alpha_e \rho_{eff}\right)}{E_s} \tag{5a}$$

by limitation

$$\varepsilon_{sm} - \varepsilon_{cm} \geq 0{,}6 \frac{\sigma_s}{E_s} \ , \tag{5b}$$

where:

s_s is the stress in the tension reinforcement assuming a cracked section;

a_e the ratio E_s/E_{cm} ;

r_{eff} $= A_s / A_{c,eff}$

$A_{c,eff}$ the effective area of concrete in tension surrounding the reinforcement of depth $h_{c,eff}$, where $h_{c,eff}$ is the lesser of 2,5(h – d), (h – x)/3, h/2 see Fig. 3;

A_s the area of the reinforcing tension steel;

k_t a factor dependent on the duration of the load

k_t = 0,6 for short term loading,

k_t = 0,4 for long term loading

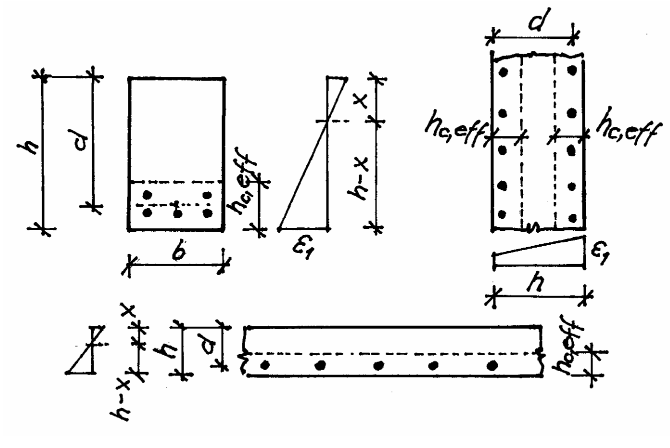


Fig. 3 Typical cases of effective tension area

In situations where bonded reinforcement is fixed at reasonably close centres within the tension zone (spacing ≤ 5(c+f/2) – see Fig. 3, the maximum final crack spacing may be calculated from the expression in [1], but modified in:

$$s_{r,max} = (k_3 c + k_1 k_2 k_4 f / r_{p,eff})\, k_5 \tag{6}$$

where

- f is the bar diameter;
- c the cover to the longitudinal reinforcement;
- k_1 a coefficient which takes account of the bond properties of the bonded reinforcement:
  k_1 = 0,8 for high bond bars,
  k_1 = 1,6 for bars with an effectively plain surface ;
- k_2 a coefficient which takes account of the distribution of strain:
  k_2 = 0,5 for bending,
  k_2 = 1,0 for pure tension,
  for cases of eccentric tension or for local areas – see [1];
- k_5 a coefficient which takes account of the fibres

$$k_5 = \frac{f_{ctk,t} - f_{ctk,eq,t,I}}{f_{ctk}}\ ; \tag{7}$$

- $f_{ctk,,t}$ the characteristic value of tensile strength of concrete in the time t.

The recommended values are k_3 = 3,4 and k_4 = 0,425.

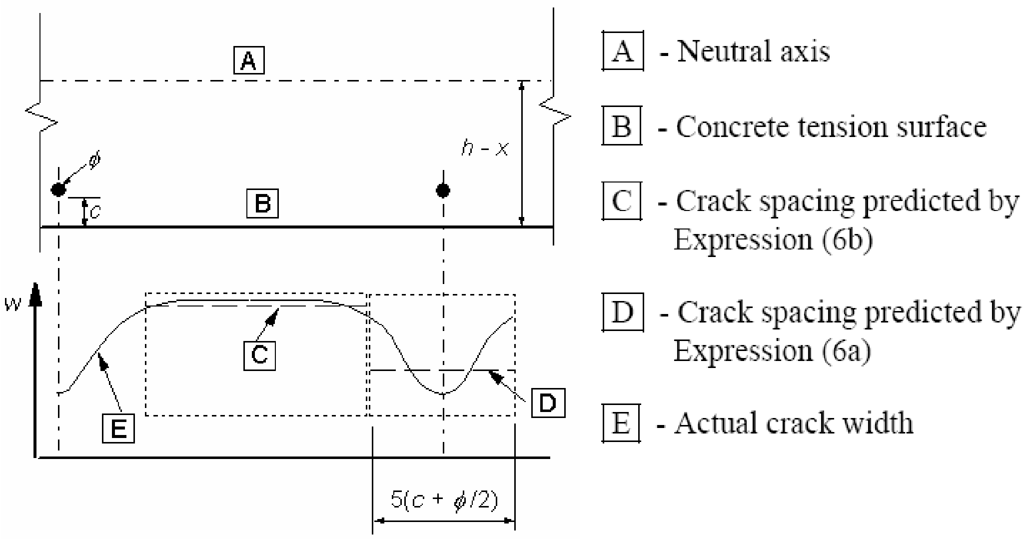


Fig. 4 Crack width w at concrete surface relative to distance from bar

Where the spacing of the bonded reinforcement exceeds 5(c+f/2) - see Fig. 3, an upper bound to the crack width may be found by assuming maximum crack spacing:

$$s_{r,max} = 1{,}3\,(h - x) \tag{6b}$$

The area A_s of the steel required it is possible determine by substitution the expressions (5a), (5b), (6a), (6b), to the equation (4). If we suppose that the spacing of the bars in cross section is

$s_t \leq 5(c+f/2)$, (7)

then we can consider for the maximum cracks spacing only the expression (6a) and for area A_s of the steel required for the limitation width of crack we consider only the expressions (5a), (6a) and (5b), (6a) to introduce to the equation (4). For A_s we receive the quadratic equation whose solution is

$$\boldsymbol{A_s} = \frac{-\boldsymbol{K_b} + \sqrt{\boldsymbol{K_b}^2 - 4 \cdot \boldsymbol{K_a} \cdot \boldsymbol{K_c}}}{2 \cdot \boldsymbol{K_a}} \quad [\mathrm{m}^2], \tag{8}$$

where a) by substitution expressions (5a) and (6b) to equation (4):

$$K_a = E_s \cdot w_k + k_3 \cdot k_5 \cdot c \cdot a_e \cdot k_t \cdot f_{ct,eff} \quad [\mathrm{N \cdot m^{-1}}], \tag{8a}$$

$$K_b = -k_3 \cdot k_5 \cdot c \cdot \left(F_s - k_t \cdot F_{ct,eff}\right) + k_1 \cdot k_2 \cdot k_4 \cdot k_5 \cdot f \cdot a_e \cdot k_t \cdot F_{ct,eff} \quad [\mathrm{N \cdot m}], \tag{8b}$$

$$K_c = -k_1 \cdot k_2 \cdot k_4 \cdot k_5 \cdot f \cdot A_{c,eff} \cdot \left(F_s - k_t \cdot F_{ct,eff}\right) \quad [\mathrm{N \cdot m^3}]; \tag{8c}$$

this is valid for the case, when ($\varepsilon_{sm} - \varepsilon_{cm}$) calculated from the expression (5a) fulfils the condition (5b),

b) by substitution expressions (5b) and (6b) to equation (4):

$$\boldsymbol{K_a} = \boldsymbol{E_s} \cdot \boldsymbol{w_k} \quad [\mathrm{N \cdot m^{-1}}], \tag{8d}$$

$$K_b = -k_3 \cdot k_5 \cdot c \cdot 0{,}6 \cdot F_s \quad [\mathrm{N \cdot m}], \tag{8e}$$

$$K_c = -k_1 \cdot k_2 \cdot k_4 \cdot k_5 \cdot f \cdot A_{c,eff} \cdot 0{,}6 \cdot F_s \quad [\mathrm{N \cdot m^3}]; \tag{8f}$$

this is valid for the case, when ($\varepsilon_{sm} - \varepsilon_{cm}$) calculated from the expression (5a) does not fulfil the condition (5b);
in the expressions (8) are

$A_{c,eff}$ = b . $h_{c,ff}$, where $h_{c,ff}$ is the lesser of 2,5(h – d), (h – x)/3, h/2 - see Fig. 3;
$F_{ct,eff}$ = $A_{c,eff}$. $f_{ct,eff}$ is the force in the crack on effective area $A_{c,eff}$;
F_s is the pension force transferred by the reinforcement;
$f_{ct,eff}$ is the tensile strength of the concrete at the time under consideration;
E_s = 200 000 MPa, modulus of elasticity of reinforcing steel;,
w_k is the crack of width.

The area of reinforcement A_s required for the limitation crack width by the full restricted intrinsic deformations of the element we can determine by consideration force in the reinforcement F_s that is equal to the force by F_{cr} given by the expression (3), by consideration (1 + $a_e \cdot r_{eff}$) = 1 and k_t = 0,4. The area of reinforcement A_s we can determine from the expression (8) - where we consider a_e = 0 and k_t = 0,4 as follow from previous.

Procedure according to DIN 1045-4

The width of crack according the recommendation given in DIN 1045-1 [2] is calculated an accordance with the expression (4), where ($\varepsilon_{sm} - \varepsilon_{cm}$) can be calculated using the expressions (5a) or (5b), but for the maximum crack spacing are there given another expressions, than can be for the fibre concrete modified in

$$s_{r,max} = \frac{f}{3{,}6\, r_{eff}} k_5 \,, \tag{9a}$$

$$s_{r,max} \leq \frac{s_s f}{3{,}6\, f_{ct,eff}} k_5 \,, \tag{9b}$$

other signs with given values shown in Chapter 2 are valid.

By the calculation of A_s to the formula (4) we will assume the expressions (5a),(5b) and (9a), (9b).

For the area A_s of the steel required for the limitation of crack width in the case of consideration expressions (5a), (9a) and (5a), (9b) we receive

$$\boldsymbol{A_s} = \frac{-\boldsymbol{K_b} + \sqrt{\boldsymbol{K_b^2} - 4 \cdot \boldsymbol{K_a} \cdot \boldsymbol{K_c}}}{2 \cdot \boldsymbol{K_a}} \,, \tag{10}$$

where a) by substitution expressions (5a) and (9a) to equation (4):

$$K_a = \frac{3{,}6 \cdot w_k \cdot E_s}{f \cdot k_5} \,, \tag{10a}$$

$$\boldsymbol{K_b} = \alpha_{\boldsymbol{e}} \cdot \boldsymbol{k_t} \cdot \boldsymbol{F_{ct,eff}} \,, \tag{10b}$$

$$\boldsymbol{K_c} = -\boldsymbol{A_{c,eff}} \cdot \left(\boldsymbol{F_s} - \boldsymbol{k_t} \cdot \boldsymbol{F_{ct,eff}}\right); \tag{10c}$$

this is valid for the case, when ($\varepsilon_{sm} - \varepsilon_{cm}$) calculated from the expression (5a) fulfils the condition (5b) and $s_{r,max}$ calculated from the expression (9a) fulfils the condition (9b);

b) by substitution expressions (5a) and (9b) to equation (4):

$$K_a = \frac{3{,}6 \cdot w_k \cdot E_s}{f \cdot k_5} \,, \tag{10d}$$

$$\boldsymbol{K_b} = \boldsymbol{F_s} \cdot \boldsymbol{k_t} \cdot \alpha_{\boldsymbol{e}} \,, \tag{10e}$$

$$\boldsymbol{K_c} = -\boldsymbol{F_s} \cdot \left(\frac{\boldsymbol{F_s}}{\boldsymbol{f_{ct,eff}}} - \boldsymbol{k_t} \cdot \boldsymbol{A_{c,eff}} \right); \tag{10f}$$

this is valid for the case, when ($\varepsilon_{sm} - \varepsilon_{cm}$) calculated from the expression (5a) fulfils the condition (5b) and $s_{r,max}$ calculated from the expression (9a) does not fulfil the condition (9b).

Further we have to consider the expressions (5b), (9a) a (5b), (9b); in this case fall of the linear term of quadratic equation, so we receive directly

a) by substitution expressions (5b) and (9a) to equation (4):

$$A_s = \sqrt{\frac{f \cdot k_5 \cdot F_s \cdot A_{c,eff}}{6 \cdot w_k \cdot E_s}} \,, \tag{11a}$$

this is valid for the case, when ($\varepsilon_{sm} - \varepsilon_{cm}$) calculated from the expression (5a) does not fulfil fulfils the condition (5b) and $s_{r,max}$ calculated from the expression (9a) fulfils the condition (9b);

b) by substitution expressions (5a) and (9b) to equation (4):

$$A_s = \sqrt{\frac{F_s^2 \cdot f \cdot k_5}{6 \cdot w_k \cdot E_s \cdot f_{ct,eff}}}\,, \tag{11b}$$

this is valid for the case, when ($\varepsilon_{sm} - \varepsilon_{cm}$) calculated from the expression (5a) does not fulfil fulfils the condition (5b) and $s_{r,max}$ calculated from the expression (9a) does not fulfil the condition (9b).

The area of reinforcement A_s required for the limitation crack width by the full restricted intrinsic deformations of the element we can determine by using expressions (12) by considering $a_e = 0$ and $k_t = 0{,}4$.

Computer program

For the design of required area A_s of the steel required for the limitation width of crack was developed the computer program. The program is available on web pages of Prof. Procházka.

Conclusions

The required area A_s of the steel required for the limitation of the width of cracks in the case of early cracks (cracks due to restricted intrinsic deformations of the element) and also later cracks (cracks deduced predominantly from loading) it is possible to design directly using the expressions derived in chapter 2 and 3.

By the consideration of the maximum spacing of cracks according to DIN 1045-1, we receive the smaller required area A_s of the steel required for the limit crack width than according to EN 1992-1-1.

Acknowledgements

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References

[1] EN 1992-1-1: Eurocode 2: Design of concrete structures – Part 1-1: General rules and rules for buildings, 2004

[2] DIN 1045-1: Tragwerke aus Beton, Stahlbeton und Spannbeton Teil 1: Bemessung und Konstruktion, Juli 2001

[3] Lohmayer, G, Ebeling, K.: Weiße Wannen einfach und sicher, Konstruktion und Ausführung wasserdurlässiger Bauwerke aus Beton, 8. überarbeitete Auflage. Verlag BAU + TECHNIK, Düsseldorf, 2007

[4] DBV-Merkblatt: Stahlfaserbbeton, Deutscher Beton -und Bautechnik Verein E. V.; Fassung Oktober 2001