APPROXIMATE METHOD OF THE INVERSE ANALYSIS OF FIBRE REINFORCED CONCRETE

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Abstract

A simple analytical method intended as an effective design tool for identification of material parameters of fibre reinforced concrete is presented. The method is based on simplified distribution of normal stresses over the cross section depth and allows the determination of material parameters from simple calculations.

Keywords: Inverse analysis, analytical formulas

1 Introduction

To reach a reliable and economic design of fibre reinforced concrete structures and structural members, their structural analyses, commonly performed by the finite element method via available computer programs, must be based on adequate material models.

It is evident that the material relations cannot be obtained directly by laboratory tests on axially loaded specimens. One of the typical laboratory tests for fibre reinforced concrete to obtain the basic material characteristics is the test of a beam loaded by two vertical forces. The standard set-up of this test is a simple supported beam loaded by two transverse forces F in thirds of the span. The bending moment is constant in the central third of the beam, shear forces appear only in the outer thirds.

Results of this laboratory test provides only a relation between deflection z and loading $F(z)$. However, for application in analyses of real structures, we need the constitutive relations describing the material properties.

2 Inverse analysis

2.1 Bending moment – curvature diagram

The first step of the inverse analysis involves derivation of the relation between curvature of deflection line of the central part of the beam and corresponding bending moment (diagram relating the bending moment M to curvature k) from the relation between the deflection and the load obtained by a laboratory test.

As mentioned above, the bending test provides only the relation between the deflection z and the load $F(z)$.

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The midspan deflection of a beam (with the rectangular cross-section of the width ***b*** and height ***h***), increasing during the load test, is possible [1] to express in terms of increasing curvature of the deflection line ***k*** and the acting loads ***F*** (taking account also the effect of shear forces in the outer regions) as

$$z = \frac{5}{8}a^2 k + \frac{F(z)a^3}{3EI} + 1.44\frac{F(z)a}{Ebh} \tag{1}$$

where ***I*** is the second moment of area of the beam cross section, ***a*** represents one third of the span length ***l***, the initial modulus of elasticity ***E*** can be determined by the initial bending response of the beam.

In such a way, the real performance of the central third of the test beam with randomly located cracks, is - as an extreme simplification – modelled by a flexural finite element with smeared cracks. With this concept the local discontinuities are distributed over some area. Hence, the constitutive behaviour of cracked fibre reinforced concrete may be modelled in terms of stress-strain relations.

This approximation may also be justified by the fact that the crack location - even in theoretically identical beams of one series - does not exhibit in the reality the same pattern in the all beams. Also, the size of the central third of test beams corresponds to sizes of finite elements commonly used in analyses of real fibre reinforced concrete structures.

For the square cross-section with side s, Eq. (1) takes the form

$$z = \frac{5}{8}a^2 k + \frac{F(z)a}{Es^2}\left(4\frac{a^2}{s^2} + 1.44\right) \tag{2}$$

This formula allows express the curvature of deflection line of the central part of the beam as

$$k = \frac{8}{5a^2}\left(z - \frac{F(z)a}{Es^2}\left(4\frac{a^2}{s^2} + 1.44\right)\right) \tag{3}$$

where ***z*** is the midspan deflection and ***F***(***z***) is the load.

The corresponding bending moment in the central part of the beam is

$$M(z) = F(z)\,a \tag{4}$$

The demanded bending moment - curvature diagram can be derived combining Eqs. (3) and (4).

2.2 Stress – strain diagram

The second step of the inverse analysis is directed to construction of the stress-strain diagram of the material that would fit the flexural behaviour described by the obtained bending moment-curvature diagram.

Various approaches based on layered models have been developed for such an inverse analysis. As these methods are well documented in the literature no attempt will be made to review them in detail here. Results of these numerical procedures are, however, available only in form of sets of numbers or graphs which are quite unsuitable for parametric studies and particularly for stochastic analyses requiring many repeated calculation runs (e.g. Monte Carlo or LHS methods). Moreover, these numerical procedures do not allow obtaining analytical expressions of the results suitable for creating engineering judgement of the nature of phenomena.

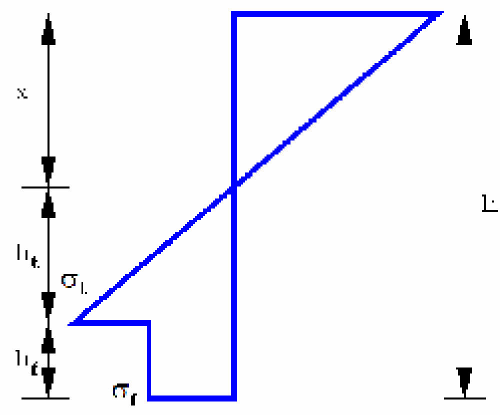

Fig. 1 Distribution of stress over the beam cross-section

The presented paper is directed to obtain results as analytical functions or formulas intended as an effective design tool, allowing easily varying individual input parameters to assess effects of these parameters. This is why we consider the simplified stress distribution along the depth of the cross section as shown in Fig. 1 - in the tension part of a cross section (where strain e_t is exceeded) distribution of stress is of a constant magnitude s_f.

The depth of a tension part of the section with a linear distribution of a stress (Fig.1) is

$$h_t = e_t / k \qquad (5)$$

maximal value of compression stress is

$$s_c = \sigma_t x / h_t \qquad (6)$$

where x is the height of the compression part of the section (Fig.1).

Equilibrium of cross section requires

$$s_f = (s_c x - s_t h_t) / 2 h_f \qquad (7)$$

By combining Eqs. (6), (7) and (8) we obtain

$$s_t (x^2 k / e_t - e_t / k) - 2 s_f (h - x - e_t / k) = 0 \qquad (8)$$

which takes a form of quadratic equation for the variable x

$$(s_t k / e_t) x^2 + 2 s_f x + 2 s_f e_t / k - s_t e_t / k - 2 s_f h = 0 \qquad (9)$$

The discriminant of Eq. (9) is

$$D = 4 (s_f - s_t)^2 + 8 \frac{\sigma_t \sigma_f k h}{\varepsilon_t} \qquad (10)$$

Roots x_1 and x_2 of Eq. (9) are given by the formula

$$x_{1,2} = \frac{-B \pm \sqrt{D}}{2A} \qquad (11)$$

where

$$A = s_t\, k \,/\, e_t\,, \quad B = 2\; s_f \tag{12}$$

Roots x_1 and x_2 are real numbers if the discriminant D is greater than zero or equal to zero. This condition is satisfied for all combination of realistic input parameters.

The variable x represents the depth of the compression part of the section (Fig.1), so the variable x must be greater than zero. Therefore, the solution obtained by the formula

$$x = \frac{-B + \sqrt{D}}{2A} \tag{13}$$

is considered in the next considerations. Eq. (13) can be rearranged to the formula

$$x = \frac{q(k)\varepsilon_t}{k} \tag{14}$$

in which the parameter $q(k)$ as a function of curvature k is given by

$$q(k) = \frac{-\sigma_f + \sqrt{\left(\sigma_f - \sigma_t\right)^2 + 2\dfrac{\sigma_t \sigma_f kh}{\varepsilon_t}}}{\sigma_t} \tag{15}$$

Stress s_f can be expressed as a fraction of the stress s_t

$$s_f = r\; s_t \tag{16}$$

where value of the parameter r is from interval < 0 ; 1 >. Formula (15) is then simplified to

$$q(k) = -r + \sqrt{(\rho - 1)^2 + \frac{2\rho kh}{\varepsilon_t}} \tag{17}$$

The resulting bending moment M in the central part of the beam is given by the formula

$$M = b\left\{\frac{1}{3}\sigma_t \frac{\varepsilon_t^2}{k^2}\left[1 + \left(q(k)\right)^3\right] + \frac{\sigma_t \rho}{2}\left[h - \frac{\varepsilon_t}{k}\left(q(k) + 1\right)\right]\left[h - \frac{\varepsilon_t}{k}\left(q(k) - 1\right)\right]\right\} \tag{18}$$

In this way, the analytical expression of the diagram relating bending moment M to curvature k is obtained; the value of bending moment M is expressed as a function of the curvature k and independent parameter r. Input parameters b, h, s_t, e_t are for every beam and material unique.

The presented analytical procedure is intended as an effective design tool. Since the diagram relating bending moment M to curvature k is expressed as an analytical formula (18), it is very easily possible to perform parametric studies to investigate the effects of input parameters and their variations. Simultaneously, we obtain an engineering judgement about fibre reinforced concrete structural behaviour.

As a simple example of application of the formula (18), results of a parametric study for the beam are plotted in Fig. 2. Values of the bending moment *M* are calculated for various values of the parameter *r* representing the effect of fibres in concrete.

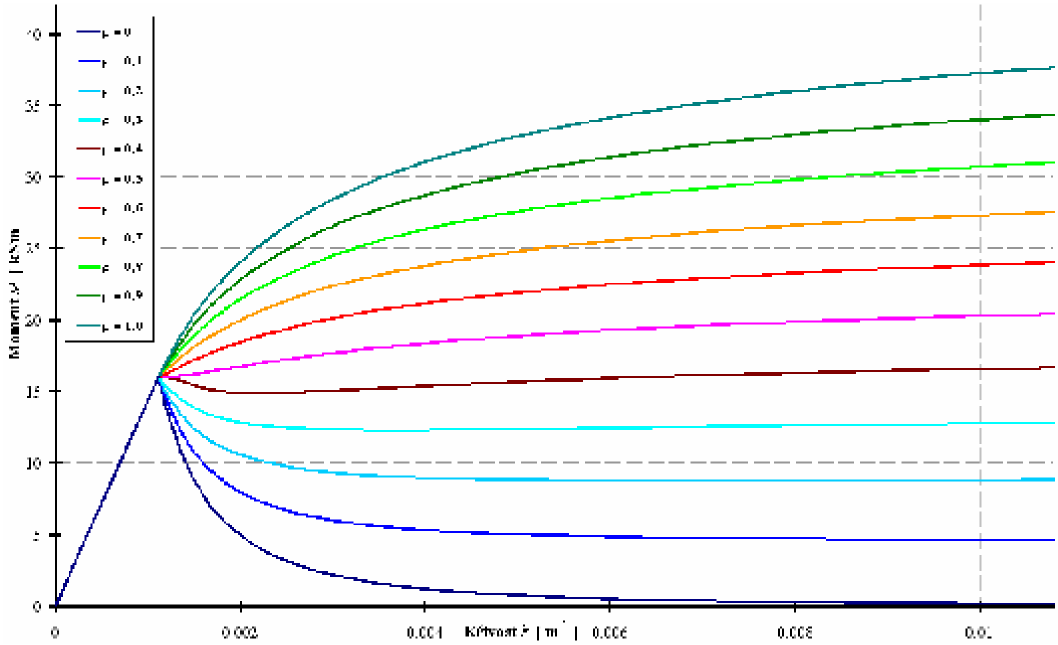

Fig. 2 Diagram relating bending moment *M* to curvature *k* for various values of parameter *r*

Several curves indicating relations of the bending moment *M* to the curvature *k* (for different values of parameter ρ) are plotted in Fig. 2. The value of the parameter *r* - expressing quantity and characteristics of fibres – very significantly influences the shape of the *M-k* diagram. Varying the *r* value, we are able to reach an optimal composition of the fibre reinforced concrete satisfying adequate mechanical as well as economic requirements. Value of parameter *r* varies theoretically between 0 (value of the stress s_f is zero – plain concrete) to 1 (value of the stress s_f is equal to value of the stress s_t). For the studied example (Fig. 2), after we reach value of the parameter *r* = 0.5, the diagram relating bending moment *M* to curvature k is progressive for all values of curvature *k*.

The intention of the inverse analysis is to identify stress-strain diagram (as it is assumed in Fig.1) – this means to find the value of parameter *r* to reach the best fit of behaviour of the tested fibre reinforced concrete beam via comparing results of tests and results of calculations.

There are two ways to obtain the best fit value of parameter *r* to approximate the fibre reinforced concrete behaviour:

We can compare diagram relating bending moment *M* to curvature *k*, derived via Eqs. (3) and (4) from a laboratory test, with the diagram calculated by the formula (18) for a variety of *r* parameter values.

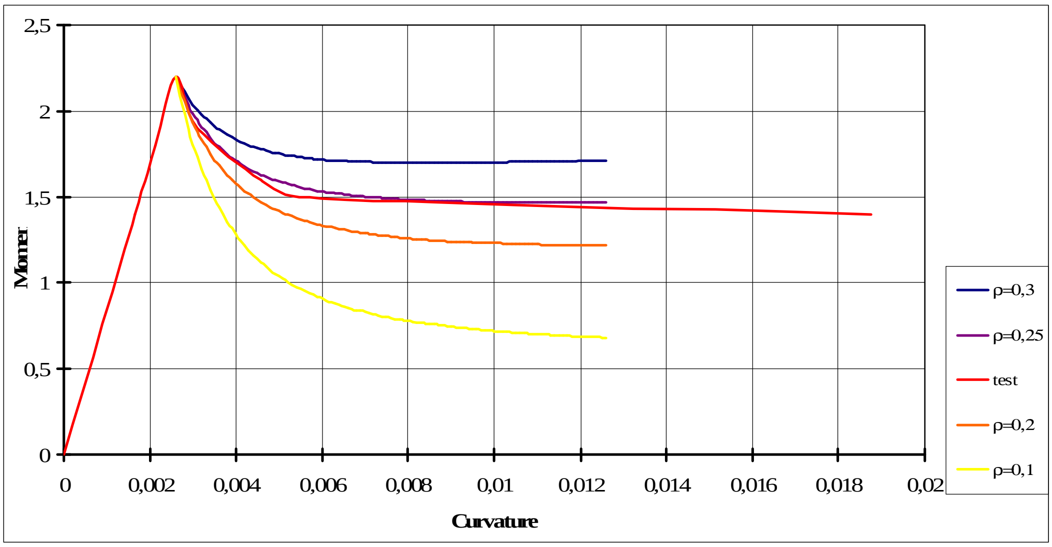

Fig. 3 Detection of parameter r from a laboratory test

Evaluation of the best fit $\tilde{r}$. parameter value can naturally be performed more sophistically, but simultaneously more laboriously. On the horizontal axis (Fig.3) we select a finite number of values of curvature k. It is obvious that accuracy of this process depends on number of such points.

The routine least squares method can be applied for determination of the best fit $\tilde{r}$ parameter value; we seek a minimum of the function

$$P(r) = \sum_{i=1}^{n} \left(\overline{M}_{k_i} - M_{k_i}(\rho) \right)^2 = \min \tag{19}$$

where $\overline{M}_{k_i}$ is bending moment corresponding to the curvature k_i based on the laboratory test, $M_{k_i}(r)$ is bending moment corresponding to the curvature k_i calculated in terms of parameter r and n is the number of selected points.

3 Conclusions

A simple analytical method intended as an effective design tool for identification of material parameters of fibre reinforced concrete is presented. The method is based on simplified distribution of normal stresses over the cross section depth and allows the determination of material parameters from simple calculations. The results are available in the form of analytical functions or formulas, allowing easily varying individual input parameters to assess effects of input parameters, to perform parametric and optimising studies and, possibly also stochastic analyses requiring many repeated calculation runs (e.g. Monte Carlo or LHS methods). Moreover, analytical expressions of the results allow also creating engineering judgement of the nature of phenomena. The intention is to reach the optimal composition of a fibre reinforced concrete (particularly the amount and characteristics of fibres) adequately fulfilling structural as well as economic requirements.

Although the method is suitable also for hand calculations it has been programmed for added convenience - the program is freely available at the web site http://concrete.fsv.cvut.cz/.

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