DESIGN OF FIBRE REINFORCED CONCRETE STRUCTURES BASED ON NONLINEAR ANALYSIS

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Abstract

The nonlinear finite element analysis can be utilized not only in research but also in the practical design of fibre reinforced concrete (FRC) structures. Behaviour of the structure under service or ultimate loading conditions can be realistically simulated using advanced computer methods. Special constitutive material models were developed for description of fracture of FRC-material in the nonlinear finite element analysis. Crack development, load carrying capacity and post-critical behaviour of engineering structures can be traced and analyzed. The nonlinear fracture analysis accounting tensile capacity of material enables to exploit reserves, which are usually neglected or diminished in codes or in linear analysis, and helps in cases which are not sufficiently supported by the codes. Recently proposed advanced design methods based on the global safety concept, utilization a newly developed ECOV-Method, stochastic LHS nonlinear analysis and structural reliability assessment of FRC structures are discussed. Practical examples of nonlinear analysis of engineering structures made from the FRC are presented.

Keywords: nonlinear analysis, structural design, fibre reinforced concrete, cracks, global safety concept

1. Introduction

The nonlinear finite element simulation is recently a well-established approach for analysis of reinforced concrete structures. Behaviour of the structure under service as well as ultimate conditions can be virtually simulated using computer methods quite realistically [1]. Nonlinear fracture analysis accounting tensile capacity of material enables to exploit reserves, which are usually neglected or diminished in codes or in linear analysis.

Behaviour and properties of fibre reinforced concrete (FRC) differ to a large extent from normal concrete. The tensile strength and especially the fracture energy are substantially larger and the form of the tensile constitutive law is different. Therefore, special material models at macroscopic level are needed for modelling of FRC-material in the numerical simulation of FRC-based structures.

2. Nonlinear finite element analysis of reinforced concrete structures

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The nonlinear numerical analysis of RC structures is based on finite element method and nonlinear material models for concrete, reinforcement and their interaction. Tensile behaviour of concrete is described by smeared cracks, crack band and fracture energy, compressive behaviour of concrete is described by damage model with hardening and softening. Reinforcement is usually modelled by truss elements embedded in two-dimensional isoparametric concrete elements. The nonlinear solution is performed incrementally with equilibrium iterations in each load step.

The finite element software ATENA is a proven tool for realistic computer simulation of damage and failure of concrete and reinforced concrete structures [2]. It contains advanced material models for concrete, reinforcement, bond etc. The tensile behaviour of concrete is modelled by nonlinear fracture mechanics combined with crack band method [3] and smeared crack concept, Fig. 1. Main material parameters for modelling of fracturing behaviour are tensile strength, fracture energy and shape of the stress-crack opening curve. For the normal concrete an exponential crack opening law derived by Hordijk [4] is applied.

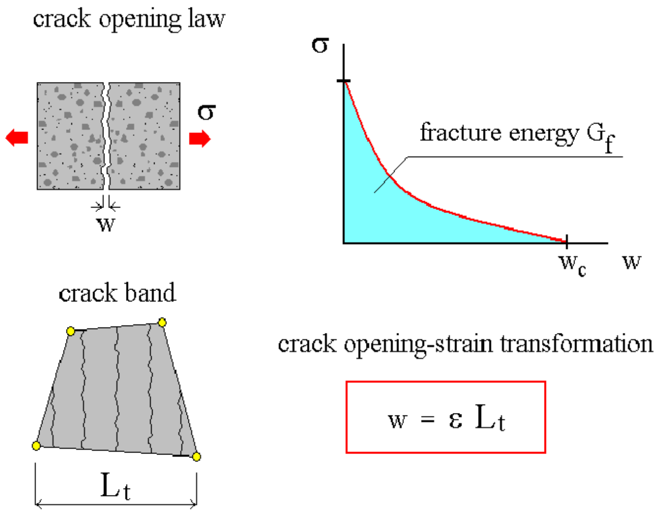

Fig. 1 Smeared crack model for tensile behaviour of normal concrete

In the smeared crack concept, the real discrete crack is substituted by a band of localized strains. The crack strain is related to the element size, which represents the crack band width Lt. Consequently, the softening law in terms of strains for the smeared model is calculated for each element individually, while the crack-opening law is preserved based on fracture energy consumption. This model is objective due to the energy formulation and its dependency on the finite element mesh is negligible.

The conventional reinforcement in standard reinforced concrete structures can be modelled as discrete bars or cables or as smeared reinforcement with given reinforcing ratio and direction. This kind of treatment of reinforcement is not well suited for description of fibres mixed in the concrete mass like in the FRC material. For this purpose special material models are introduced.

3. Constitutive models for fibre reinforced concrete

In the tensile behaviour of FRC the tensile strength and fracture energy are substantially larger than in normal concrete. As the first approach the exponential crack opening law by Hordijk with adjusted parameters can be used for FRC modelling, but the form of the tensile constitutive law doesn't correspond well to the behaviour of FRC material. Therefore, material laws with special forms of the tensile descending branch more suitable for FRC were formulated and implemented into ATENA.

Two models are designed especially for steel fibre reinforced concrete (SFRC). They are derived from plane stress material law SBETA for plain concrete [5]. If the fracture energy is known, an objective material law based on the crack band approach (Fig. 2, left) can be used. After cracking, the tensile stress drops to certain fraction of the tensile strength. The ultimate crack width w_c and slope of the linear descending branch are calculated from the fracture energy G_f and the final stress level f_2. Parameters of this model are tensile strength f_t, fracture energy G_f, and relative values c_1 and c_2 of the tensile stress levels f_1 and f_2.

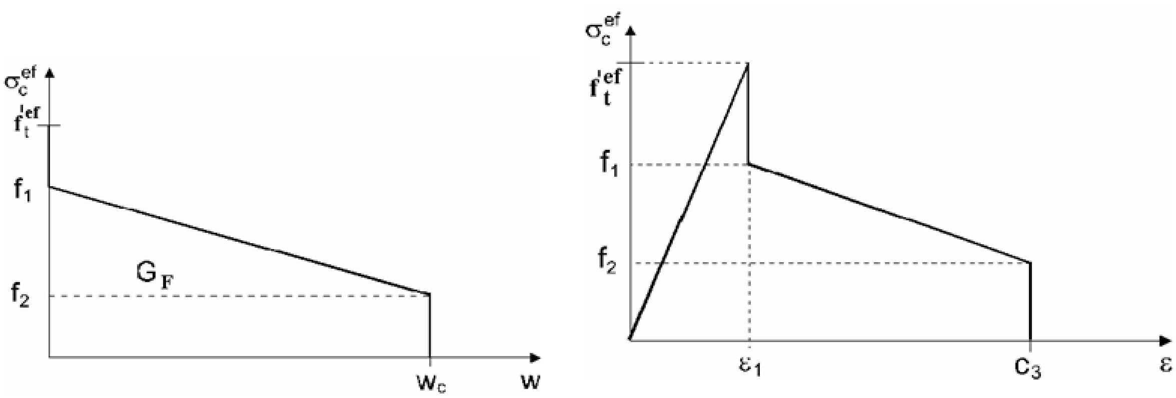

Fig. 2 Steel fibre reinforced concrete model based on fracture energy (left) and based on local strain (right)

In practical cases the fracture energy value is often difficult to evaluate since in tests it is a hard task to follow the long-persisting descending branch until the zero tensile stress. In such a case a local formulation of the tensile material law is available (Fig. 2, right). It is similar to the previous model but it is formulated directly in terms of strains and does not employ the fracture energy and crack band approach. Model parameters c_1 and c_2 are relative values of the tensile stress levels f_1 and f_2 related to the tensile strength f_t, and c_3 is the prescribed ultimate strain.

The most sophisticated and most general model of FRC material represents an extension to the fracture-plastic constitutive law [6] called CC3-User model. It describes the tensile behaviour according to the material response measured in tests point-wise in terms of the stress-strain relationship (Fig. 3). The first part of the diagram is the usual stress-strain constitutive law. After exceeding the localization strain ε_{loc} the material law assumed for the characteristic crack band width L_{ch} is adjusted to the actual crack band width L_t.

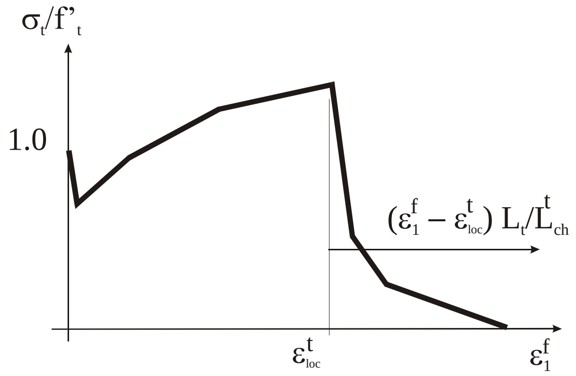

Fig. 3 User defined tensile behaviour

All these models were successfully applied in numerical simulations of experimental specimens. The FRC material can be also combined with the conventional reinforcement.

4. Safety formats in structural design

Engineers often use nonlinear analysis while making assessment of old existing structures or when designing new ones. This evolution is supported by rapid increase of computational power as well as by new capabilities of the available software tools for numerical simulation of structural performance. The nonlinear analysis is also a useful tool by utilization of modern materials like fibre reinforced concrete.

The code provisions on the other hand provide very little guidance how to use the results of a nonlinear analysis for structural assessment or design. The safety formats and rules that are usually employed in the codes are tailored for classical assessment procedures based on beam models, hand calculation or linear analysis and local section checks. Certain national or international codes have already introduced new safety formats based on overall/global safety factors to address this issue. Such codes are, for instance, German standard DIN 1045-1 or Eurocode 2 EN 1992-2.

Standard assessment procedure based on partial safety factors usually involves the following steps:

1. Linear elastic analysis of design actions E_d:

$$E_d = \gamma_{S1} S_{n1} + \gamma_{S2} S_{n2} + ...\gamma_{Si} S_{ni} \tag{1}$$

2. Design resistance (section) R_d is calculated using design values of material parameters:

$$R_d = r(f_d, ...),\ f_d = f_k / \gamma_m \tag{2}$$

3. Safety check is performed by design condition:

$$E_d < R_d \tag{3}$$

Partial safety in local sections is ensured. However, the probability of failure, i.e. the probability of violation of the design criteria (3) is not known.

In the above outlined procedure, the nonlinear analysis should be applied in step 1) to replace the linear analysis. Following the current practice an engineer will continue to steps 2), 3) and perform the section check using the internal forces calculated by the nonlinear analysis. But if the design values for material parameters are used in the nonlinear analysis, then very unrealistic, i.e. strongly degraded, material is assumed. In statically indeterminate structures, this may result in quite unrealistic redistribution of forces, which may not be necessary on the conservative side. Furthermore, in the nonlinear analysis material criteria are always satisfied implicitly by the employed constitutive laws. Therefore, it does not make sense to continue to step 3) and perform section checks. Instead, a global check of safety should be performed on a higher level and not in local sections. This is the motivation for the introduction of new safety formats for nonlinear analysis in structural design.

5. Global safety based on nonlinear analysis

The aim is to extend the existing safety format of partial factors and make it compatible with the nonlinear analysis. First we introduce a new design variable of resistance:

$$R = \boldsymbol{r}\,(f, a, ..., S) \tag{4}$$

Resistance represents a limit state. In a simple case this can be a single variable, such as loading force, or intensity of a distributed load. In general this can represent a set of actions including their loading history. We want to evaluate the reliability of resistance, which is effected by random variation of basic variables *f* - material parameters, *a* – dimensions, and possibly others.

The resistance is determined for a certain loading pattern, which is here introduced by the symbol of actions *S*. It is understood that unlike material parameters and dimensions, which enter the limit state function ***r*** as basic variables, the loading is scalable, and includes load type, location, load combination and history. It is the objective of the resistance *R* to determine the loading magnitude for given loading model.

Random variation of resistance is described by a statistical distribution characterized by following parameters: R_m - mean value of resistance, R_k - characteristic value of resistance, , i.e. 5% percentile of the resistance, R_d - design value of resistance.

The design condition is defined in analogy with partial safety factor method by Eq. (3)

In general, E_d and R_d represent set of actions and the limit state is a point in a multi-dimensional space, respectively. Resistance scaling factor k_R will describe safety factor with respect to the considered set of design actions. In the simplified form, considering one pair of corresponding components it can be described as:

$$k_R = \frac{R}{E_d} \tag{5}$$

Then, the design condition (3) can be rewritten as:

$$\gamma_R < k_R \tag{6}$$

where γ_R is required global safety factor for resistance. Factor k_R can be used to calculate the relative safety margin for resistance

$$m_R = k_R - 1 \tag{7}$$

The task now remains to determine the design resistance R_d. This can be done for example by the ECOV method, proposed by the author in [7].

6. ECOV method

This method is based on the idea, that the random distribution of resistance, which is described by the coefficient of variation V_R, can be estimated from mean R_m and characteristic values R_k. The underlying assumption is that random distribution of resistance is according to lognormal distribution, which is typical for structural resistance. In this case, it is possible to express the coefficient of variation as:

$$V_R = \frac{1}{1.65} \ln\left(\frac{R_m}{R_k}\right) \tag{8}$$

Global safety factor γ_R of resistance is then estimated as:

$$\gamma_R = \exp(\alpha_R \beta V_R) \tag{9}$$

where α_R is the sensitivity (weight) factor for resistance reliability and β is the reliability index. According to Eurocode 2 EN 1991-1, typical values are $\beta = 4.7$ (for one year) and $\alpha_R = 0.8$. In this case, the global resistance factor is:

$$\gamma_R \cong \exp(-3.76\, V_R) \tag{10}$$

and the design resistance is calculated as:

$$R_d = R_m / \gamma_R \tag{11}$$

The mean and characteristic values R_m , R_k can be estimated using two separate nonlinear analyses using mean and characteristic values of input material parameters, respectively.

$$R_m = r(f_m, ...),\ R_k = r(f_k, ...) \tag{12}$$

The method is general and reliability level β and distribution type can be changed if required. The advantage of this approach is that the sensitivity to individual parameters such as for instance steel or concrete strength can be estimated.

7. Probabilistic analysis

Probabilistic analysis is a general tool for safety assessment of reinforced concrete structures, and thus it can be applied also in case of nonlinear analysis of FRC structures [8]. In the full probabilistic approach the structural resistance R_d is calculated by means of the probabilistic nonlinear analysis.

Uncertainties in FRC-materials are of extreme importance since the scatter of experimental response is generally much higher comparing conventional concrete. The classical statistical and reliability approach is to model material parameters as random variables with prescribed distribution function. The stochastic response requires repeated analyses of the structure with these random input parameters, which reflects randomness and uncertainties in the input values.

In this approach the resistance function *r (**r**)* is represented by nonlinear structural analysis and loading function *s (**s**)* is represented by action model. Safety can be evaluated by the reliability index β, or alternatively by failure probability P_f taking into account all uncertainties due to random variation of material properties, dimensions, loading, and other.

Probabilistic analysis based on numerical simulation include following steps:

1) Numerical model based on non-linear finite element analysis. This model describes the resistance function *r (**r**)* and can perform deterministic analysis of resistance for a given set of input variables.

2) Randomization of input variables (material properties, dimensions, boundary conditions, etc.). This can also include some effects of actions, which are not in the action function *s (**s**)* (for example pre-stressing, dead load etc.). Random properties are defined by random distribution type and its parameters (mean standard deviation, etc.). They describe the uncertainties due to statistical variation of resistance properties.

3) Probabilistic analysis of resistance and action. This can be performed by stratified method of Monte Carlo-type of sampling, such as LHS sampling method. Results of this analysis provide random parameters of resistance and actions, such as mean, standard deviation, etc. and the type of distribution function for resistance.

4) Evaluation of safety using reliability index β or probability of failure.

Probabilistic analysis can be also used for determination of design value of resistance function *r (**r**)* expressed as R_d. Such analysis involves the steps 1) to 3) above and R_d is determined for required reliability β or failure probability P_f .

8. Application examples

Two practical examples of nonlinear analysis of engineering structures made from the FRC (see also [9, 10]) – railroad plates and pre-cast tunnel segments – are presented.

The rail carrying plates have been analyzed by means of nonlinear FE-method (ATENA software). The main investigated variants of the plate have been made from concrete with conventional reinforcement, and from fibre reinforced concrete with 35 kg fibres in 1 m^3 of concrete. For modelling of FRC the material model with fracture energy based descending branch (Fig. 2, left) has been employed. The used material parameters represented mean material properties. The FRC structure showed about half crack width against the conventional one. Comparison of cracks patterns in both plates is shown in Fig. 4. The results from analyses (Fig. 5) showed that the load carrying capacity as well as serviceability of both main alternatives (RC and FRC with 35 kg fibres) is similar and sufficient. The global safety factor against the service load has been around 5. But the labour demand and steel consumption is most favourable in the case of FRC.

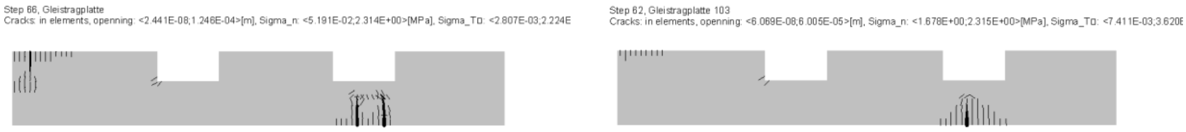

Fig. 4: Crack patterns in railroad plates at peak load, left: RC plate, max. crack width 0.12 mm, right: FRC plate, right: max. crack width 0.06 mm

During plate production it was important to check and confirm the material properties (fracture energy, tensile strength, shape of the tensile descending branch) considered in the nonlinear analysis. After concreting the plate and material test verification of the numerical results have been made based on the measured material properties.

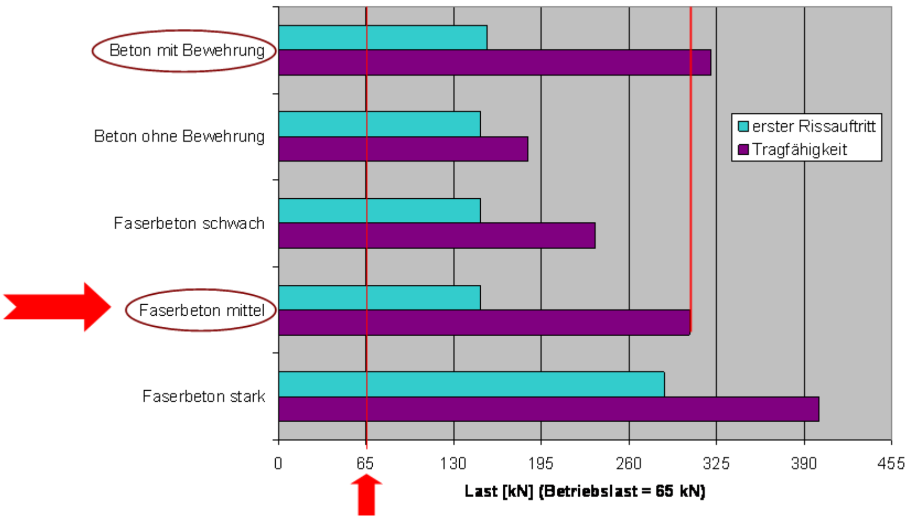

Fig. 5: Comparison of results – load carrying capacity of the plate made from in various materials

Response and ultimate load carrying capacity of pre-cast tunnel tubings used in tunnel boring method (TBM) was investigated. These tubings are usually designed and produced from reinforced concrete with steel rebars. Such tubings were tested in Klokner institute of CTU in Prague until destruction; nonlinear numerical model of the tests of RC tubing was investigated to support the experiments. An alternative design by fibre reinforced concrete without rebars was investigated only numerically. Selected results – crack patterns, comparison of the response curves and ductility – are presented in following figures. The behaviour of FRC structure was found to be superior to RC tubing. Crack patterns shown in the Fig. 6 exhibits larger local crack width for the RC tubing. In the SFRC model the cracks are less localized and they are formed in a wider band; crack opening is lower. Resistance of the SFRC specimens against the acting load is higher and the post peak behaviour is more ductile (Fig. 7). The alternative design by SFRC has better performance, and in the same time it can save up time and reduce labour.

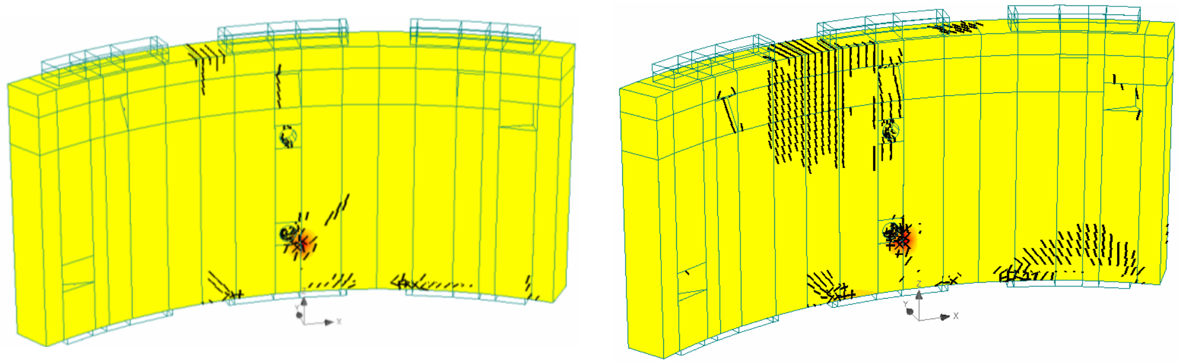

Fig. 6: Crack patterns for RC tubing (left) and FRC tubing (right)

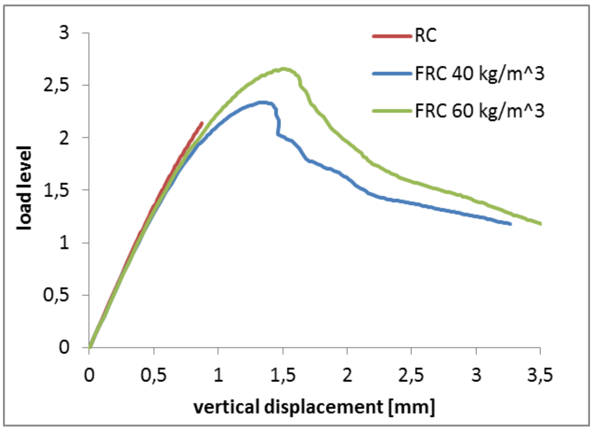

Fig. 7: Load-displacement diagram from numerical simulation of tubing – comparison of reinforced concrete and fibre reinforced concrete

RC = standard reinforced concrete with mild reinforcing steel rebars,

FRC 40 or 60 kg/m^3 = fibre reinforced concrete without rebars, with content of 40 or 60 kg of steel fibres in one cubic meter of concrete mixture

9. Conclusions

In the fibre reinforced concrete the tensile behaviour is dominating. For that reason the potential profit from the nonlinear analysis of FRC-based structures is much higher than in standard reinforced concrete structures. Therefore, advanced material models for numerical simulation of fibre reinforced concrete were developed. Sophisticated techniques for accounting uncertainties and randomness are available. The described methodology is implemented into integrated software tools for nonlinear analysis of FRC-based structures, which can be utilized in practice. This is documented on selected examples. Recently proposed advanced design methods based on the global safety concept, utilization a newly developed ECOV-Method, stochastic LHS nonlinear analysis and structural reliability assessment supports using the nonlinear analysis in design of fibre reinforced structures.

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