FIBRE CONCRETE 2011
Prague, 8th – 9th September 2011

# OPTIMUM DESIGN OF CONVENTIONAL SFRCs AND THEIR PERFORMANCE CLASSES

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## Abstract

*For determining the performance classes of steel fiber reinforced concretes (SFRCs), their equivalent flexural strengths were investigated according to both Serviceability Limit State (SLS) and Ultimate Limit State (ULS). For a certain concrete class, the equivalent flexural tensile strengths depend on both the fiber volume fraction and the fiber aspect ratio. Performance classification of SFRCs can be made according to the parameters of concrete strength, the volume fraction and the aspect ratio of steel fibers. In order to obtain optimum solutions; equivalent flexural strengths, splitting tensile strength, fracture energy were maximized and cost of fibers were minimized simultaneously. Thus, numerical optimization was used to optimize any combination of either factors or responses. For the optimum mix design of SFRCs, three-level factorial experimental design and Response Surface Method were used.*

**Keywords: Steel fiber, concrete performance class, fiber aspect ratio, equivalent flexural strength, serviceability limit state, ultimate limit state.**

## 1. Introduction

The brittleness of concrete increases with an increase in its strength. In other words, the higher the strength of concrete, the lower is its ductility. This inverse relation between strength and ductility is a serious drawback and limits the use of high strength concrete [1]. Therefore, improving the ductility of concrete becomes a major problem for high strength concrete. In concrete, the enhancement of the ductility can be realized by using steel fibers. The use of steel fibers in concrete greatly increases its energy absorption and ductility [2]. The main contribution of steel fibers to concrete can be seen after matrix cracking. If a proper mixture is designed, after the matrix cracking, randomly distributed short fibers in the matrix act as crack arresters by bridging mechanism, undergo a pull-out process, delay crack formation and limit crack propagation [3,4]. Debonding and pulling out of fibers

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from the matrix require more energy; therefore, a substantial increase occurs in toughness [5]. Fibre type, aspect ratio (length/diameter), volume fraction, orientation of fibers in the matrix and pull-out resistance of fibers, as well as matrix properties influence the performance of SFRC [6-8].

SFRC has a wide range of applications such as; pavements and overlays, industrial floors, precast elements, hydraulic and marine structures, repairing and retrofitting of reinforced concrete structures, tunnel linings and slope stabilization works [9]. Especially, major applications of SFRCs are pavements and other types of slabs and decks [10]. Small industrial floors are mainly subjected to impact and other mechanical loads, however, large industrial floors and large airport runways have to resist shrinkage and thermal cracking, as well as the mechanical loads [11].

In Germany, the DBV-Merkblatt code [12] is used for the design of floor slabs of SFRC. Two different methods of design are available; in the first one, the elastic theory is used while the second takes the energy absorption capacity of SFRC into consideration. Other energy-based design approaches have been proposed, for example, by Moens and Nemegeer [13] using toughness ratings and ultimate flexural strengths, and by Falkner et al. [14] through the equivalent flexural strength obtained from a beam test.

The main objective of this work is to determine the performance classes in conventional SFRCs from the equivalent flexural tensile strength point of view. In order to obtain optimum solutions, equivalent flexural tensile strengths for both Serviceability Limit State (SLS) and Ultimate Limit State (ULS), splitting tensile strength and fracture energy were maximized, and fiber content and cost of the mixture were minimized simultaneously. For the optimum mix design of SFRCs, three-level factorial experimental design and Response Surface Method were used.

## 2. Optimum Design of Conventional SFRCs

Effects of the aspect ratio (L/d) and volume fractions of steel fibers ($V_f$) on the fracture properties of normal and high strength concretes were investigated by Yalcin [15] using three groups of experiments in bending by measuring the fracture energy and ductility. In the first group of the experiments, the effect of the same type of hooked end steel fibers on the mechanical and performance properties of concrete having two different concrete classes was investigated. The steel fibers of normal tensile strength have the same aspect ratio (L/d=80), but four different volume fractions ($V_f$). Volume fractions of steel fibers were varied between 0.19% and 0.58%, in steps of 0.13%. Thus, in this group, mixtures including two control concretes with water-cement ratios of 0.45 and 0.65, and eight mixtures of SFRCs, therefore a total of ten mixtures were produced. In the second group, the effect of aspect ratio of the steel fiber on the mechanical and performance properties of steel fiber-reinforced concrete was investigated. The normal strength steel fibers having the aspect ratios of 80, 65 and 55 were utilized. In this group, nine concrete mixtures with steel fibers, and one control concrete were cast. In the third group of the experiments; the aspect ratio was 80 for all fibers. Lengths of these high strength steel fibers were; 60 mm, 40 mm and 30 mm, and they added in hybrid form to the mixtures having water-cement ratios of 0.32, 0.44 and 0.75. Volume fractions of these high strength hybrid steel fibers were; 0.27%, 0.50% and 0.73%. Thus, twelve concrete mixtures were produced for this group.

The load versus displacement curve for each beam of the mixtures was obtained by recording the average of three measurements taken at the mid span.

### 2.1 Test Procedure

Standard strength tests were conducted in accordance with European Standards (EN 206-1 and EN 12390). As seen in Figure 1a four-point bending tests were performed on the beams of 150x150x750 mm size. For the plain concretes, the displacement rate at the mid-span of the beams was kept constant at 0.02 mm/min. The beams with steel fiber, however, were tested at the displacement rate of 0.05 mm/min up to a displacement of 0.5 mm, and then at 0.1 mm/min up to a 5 mm displacement. The load was applied by an Instron 5500R closed-loop testing machine of 100 kN capacity, and displacements were measured simultaneously by using three linear variable displacement transducers (LVDTs). The load versus displacement curve for each beam was obtained by recording the average of three measurements taken at the mid span. The load-displacement curves were used for evaluating the equivalent flexural strengths of both SLS (Serviceability Limit State) and ULS (Ultimate Limit State). As seen in Figure 1b, the area under the load versus displacement at mid span curve was described as a measure of the energy required for each displacement.

Characteristic equivalent flexural strength can be calculated as follows:

$$f_{eq} = \frac{T_i}{\delta_i} \cdot \frac{S}{bh^2} \tag{1}$$

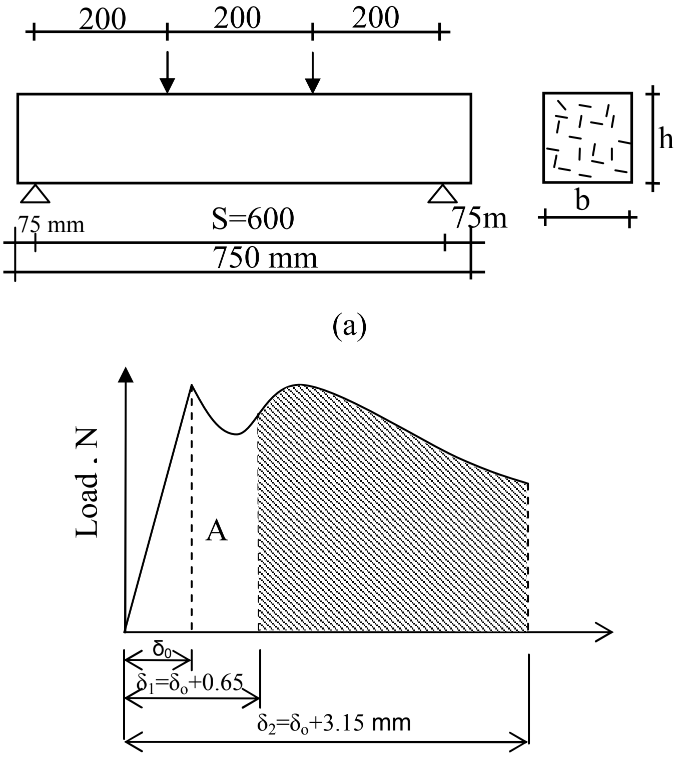
where $T_i$ is the area under load versus displacement curve for SLS or ULS, $\delta_i$ is the corresponding displacement for each limit state; bxh (150x150 mm) and $S$ (600 mm) are the cross-section of the beam and the length of the span, respectively.

According to DBV (German Concrete Society), equivalent flexural strengths of steel fiber reinforced concrete are given as indicated in Table 1 [12]. In this table and in Figure 1, $\delta_0$ shows the displacement at the first crack.

Tab.1. Deformation regions for SFRCs [12].

| Deformation Region | Limit State | Displacement |
|---|---|---|
| I (small deformation, A) | SLS | $\delta_1 = \delta_0 + 0.65$ |
| II (large deformation, A+B) | ULS | $\delta_2 = \delta_0 + 3.15$ |

The specimens were cast in steel moulds and compacted on a vibration table. All the specimens were demoulded after about 24 hours, stored under wet burlap at 20°C until 28 days of age, and were then air-cured in the laboratory until their testing date at 56 days. The dimensions of the beams, prepared for four point bending tests, were 150x150x750 mm. At least four beam specimens from each concrete mixture were tested. For each mixture, three cylinders, 150 mm in diameter and 300 mm in height, were used for compressive strength and modulus of elasticity tests. Six disc specimens, 150 mm in diameter and 60 mm in height, were prepared for the splitting tests.



(a)

(b)

Fig.1. Four point bending test specimen (a), calculation of equivalent tensile strength for each deformation region (b) [12].

## 2.2 Complete load-displacement curves under bending

Based on the second group of Yalcin's experiments [15], typical load-displacement curves for the mixture containing fibers with the aspect ratio of 80 are shown in Figure 2. These curves obtained in this study were used for evaluating the equivalent flexural strengths for both SLS and ULS. The area under each curve was indicated to be a measure of the fracture energy of the material. It can clearly be seen that fracture energy increases as the fiber volume fraction of steel fiber increases. As seen in the figure, after the formation of the first crack, except the mixture with the steel fiber volume fraction of 0.32%, the progress of strain hardening in the ascending branch of the curve is a typical indication of high performance cement based composites. Similar results were obtained for the mixtures with the other aspect ratios.

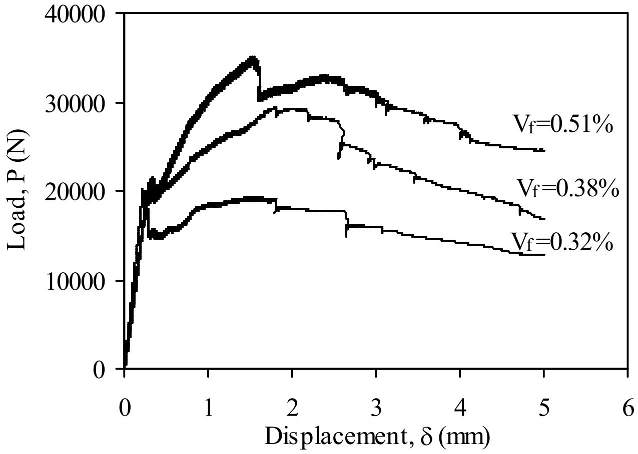
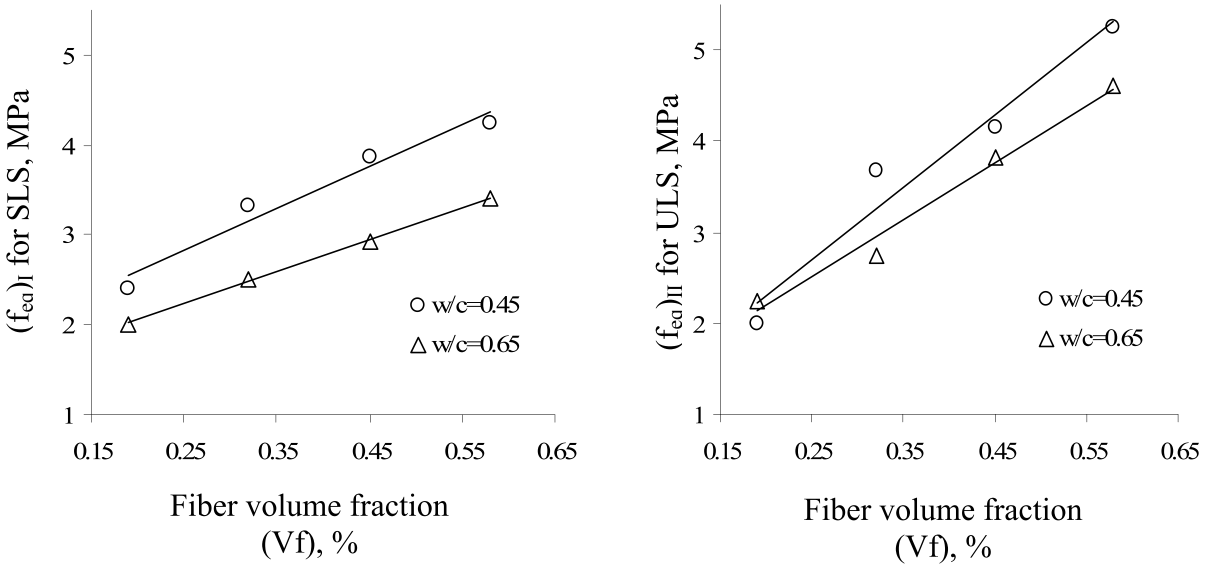


Fig.2. Typical load versus displacement at the midspan-curves for the second group of mixtures containing fibers with the aspect ratio of 80 [15].

One of the major roles of fiber in concrete is to provide an increase in the fracture energy. The results reported by Yalcin et al. [15] here are also based on the area under the complete load-deflection curve up to a specified deflection. The specified deflections for SLS and ULS are $\delta_0 + 0.65\,\mathrm{mm}$ and $\delta_0 + 3.15\,\mathrm{mm}$, respectively. In Figure 2, the cut-off point was chosen as 5mm deflection. It is seen from these curves that, the energy at this deflection (i.e. 5mm), however, is not totally dissipated. SFRCs allow obtaining high values of equivalent flexural strengths and high ductility, depending on their strengths and the volume fraction of fibers used. The increase in equivalent strengths for both SLS and ULS is because of the high energy of fiber pull-out and fiber debonding in the fracture process.

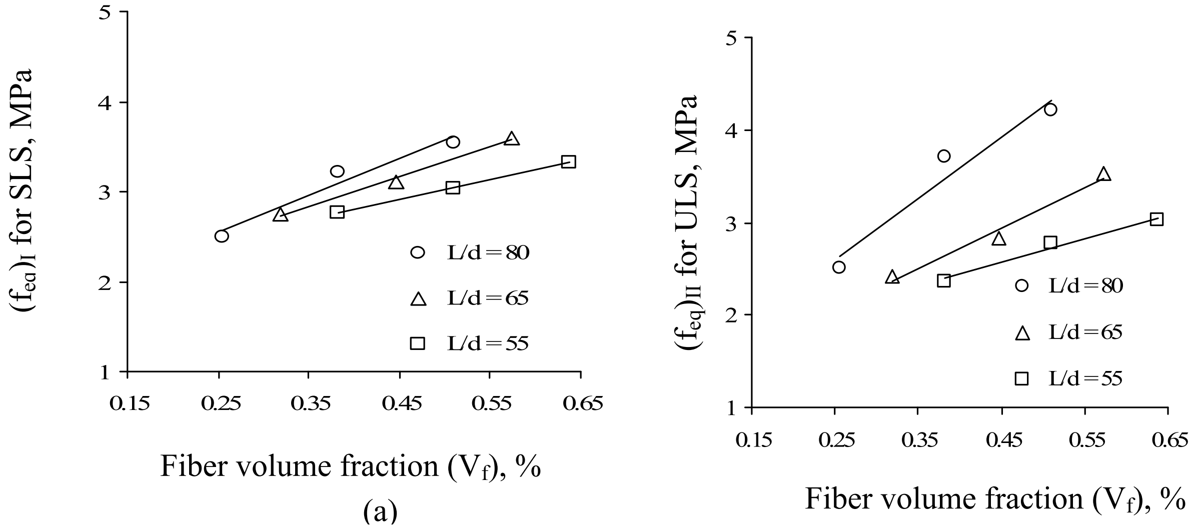
Figure 3 shows effects of concrete strength and fiber content on the equivalent flexural strengths for serviceability and ultimate limit states. For a certain volume fraction of hooked end steel fibers, the equivalent strength $(f_{eq})_I$ increases significantly as the water-cement ratio of SFRC decreases. However, in SFRCs with the water/cement ratio of 0.65, equivalent strength $(f_{eq})_{II}$ increases slightly with increasing the steel fiber volume fraction. It should be noted that these experimental results are valid for normal strength matrix and low carbon steel fiber; which has yield strength of 1100 MPa. It can be concluded that the ability of the beam to absorb energy is substantial, even if the cut-off point is taken at the specified deflections of $\delta_0$ + 0,65 and $\delta_0$ + 3,15 mm. Hence, it can be concluded that the results obtained give a clear picture of how a quasi-brittle concrete transforms into a ductile composite with the addition of steel fibers [16, 17].

Based on the test results obtained by Yalcin et al. [18], the performance classes of SFRCs for both small and large deformations (i.e. SLS and ULS), can be given. For example, the performance class of SFRC with water-cement ratio of 0.45 and fiber volume fraction of 0.19% can be shown as, C 40/50 F 2.39/2.00. Similarly, for the mixture of SFRC with water/cement ratio 0.65 and $V_f$ = 0.19%, its performance class becomes C 30/37 F 2.01/2.25 [18].



(a) (b)

Fig 3. Equivalent flexural strength versus fiber volume fraction curves for SLS (a), Equivalent flexural strength versus fiber volume fraction curves for ULS (b) [18].



(a) (b)

Fig 4. Equivalent flexural strength versus fiber volume fraction curves for SLS (a), equivalent flexural strength versus fiber volume fraction curves for ULS (b) (w/c=0.55) [19].

Figure 4 shows the effects of aspect ratio and fiber content on the equivalent strengths for SLS and ULS. For a certain volume fraction of hooked end steel fibers, the equivalent flexural strengths ($f_{eq-I}$, $f_{eq-II}$) increase significantly, as the aspect ratio of SFRC increases.

In SFRCs with the aspect ratio of 80, $f_{eq\text{-}II}$ increases rapidly with the increasing steel fiber volume fraction. It should be noted that these experimental results are valid for normal strength matrix and low carbon steel fiber; which has yield strength of 1100 MPa.

In the second group of their experiments obtained by Yalcin [15], the effect of aspect ratio of the steel fiber on the mechanical and performance properties of steel fiber-reinforced concrete was investigated. The normal strength steel fibers having the aspect ratios of 80, 65 and 55 were utilized. In this group, nine concrete series with steel fibers, and one control concrete were cast.

It can be concluded that the capability of the beam to absorb energy is substantial, even if the cut-off point is taken at the specified displacements of $\delta_0$ + 0.65 and $\delta_0$ + 3.15 mm. Figures 4a and 4b show the effects of fiber content on the equivalent flexural tensile strengths for both SLS and ULS for three different aspect ratios. As seen in these figures, for a certain volume fraction of hooked end steel fibers, equivalent flexural tensile strength ($f_{eq\text{-}I}$ or $f_{eq\text{-}II}$) increases significantly as the aspect ratio of steel fiber increases. For a certain concrete class it is seen that, the fiber content and aspect ratio are the main variables in determining the performance classes of SFRCs [14, 20].

Based on the test results obtained by Yalcin [15], the performance classes of SFRCs for both small and large deformations (i.e. SLS and ULS) can be given. For example, the performance class of SFRC with water/cement ratio of 0.55, fiber volume fraction of 0.51% and aspect ratio of 80 can be denoted by C35/45 F 3.54/4.21. Similarly, the performance class of the mixture (L/d=65, $V_f$=0.45% and w/c = 0.55) can be said to be C35/45 F 3.11/2.82.

## 3. Optimization

A multi-objective optimization problem is solved by using the single composite response (D) given in Equation 2, which is the geometric mean of the individual desirability function [21]. The desirability approach involves transforming each estimated response, $d_i$, into a dimensionless utility bounded by $0<d_i<1$ [22].

$$D = (d_1 \times d_2 \times .......... .\times d_n)^{\frac{1}{n}} = \left( \prod_{i=1}^{n} d_i \right)^{\frac{1}{n}} \tag{2}$$

where n is the number of response included in the optimization.

After building the regression models, all independent variables are varied simultaneously and independently in order to optimize the objective functions [17]. For the second group of experiments, composite desirability (D) for this multi objective optimization is shown in Figures 5 and 6. Here the optimal values of design variables are L/d=72.2, $V_c$=40.9 kg/m$^3$, and cost of mixture is 1.97 unit.

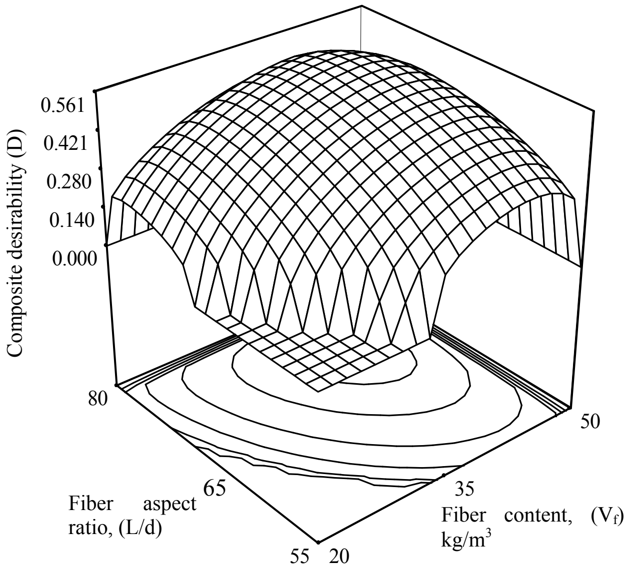


Fig. 5. Response surface plot of the composite desirability (D) for group 2 when $f_{eq\text{-}I}$, $f_{eq\text{-}II}$, $f_{sp}$ and $G_f$ are maximized, fiber content ($V_c$) and cost of mixture are minimized simultaneously [19].

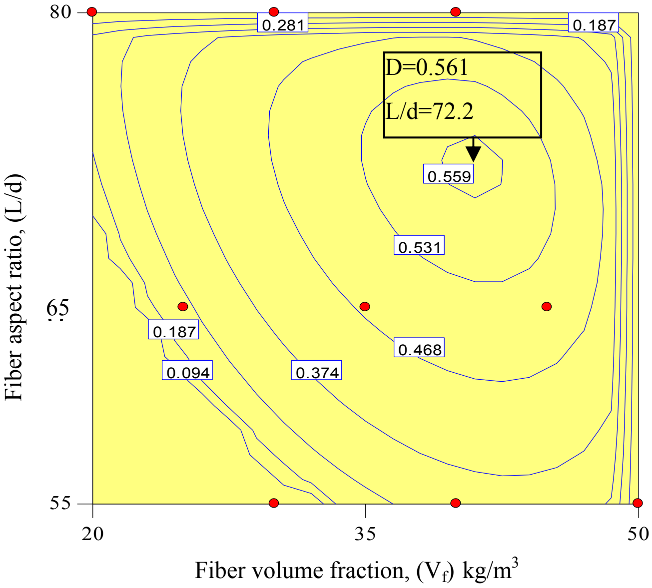


Fig. 6. Contour plot of composite desirability (D) for group 2 when $f_{eq\text{-}I}$, $f_{eq\text{-}II}$, $f_{sp,}$ and $G_f$ are maximized, fiber content ($V_c$) and cost of mixture are minimized simultaneously [19].

Tab. 2. Optimum values of three groups [19]

| Factors and responses | | 1st group | 2nd group | 3rd group |
|---|---|---|---|---|
| Factors | Water/Cement ratio (%) | 49,84 | - | 46.56 |
| | L/d | - | 72.24 | - |
| | $V_f$, kg/m$^3$ | 32 | 40.92 | 41.23 |
| Responses | Unit cost | 1.83 | 1.97 | 2.73 |
| | SLS, $f_{eq}$ I, MPa | 3.38 | 3.51 | 3.45 |
| | ULS, $f_{eq}$ II, MPa | 3.81 | 3.80 | 4.96 |
| | $f_{sp}$ , MPa | 5.05 | 4.6 | 6.33 |
| | Fracture energy, kN/m | 5 | 4.56 | 6.06 |
| | Composite desireability (D) | 0.463 | 0.561 | 0.502 |

The optimum values of the third group experiments obtained by Yalcin [15] are also shown in Table 2. In the third group of his experiments; the aspect ratio was 80 for all fibers. Lengths of these high strength steel fibers were: 60 mm, 40 mm and 30 mm, and they added in hybrid form to the mixtures having water-cement ratios of 0.32, 0.44 and 0.75. Volume fractions of hybrid steel fibers were: 0.27%, 0.50% and 0.73%. Thus, twelve concrete mixtures were produced for this group.

## 4. Performance Classes for SFRCs

Based on the three groups of experiments and within the limits of the work, performance classes of SFRC can be determined according to both SLS and ULS. The two different examples are shown in Table 2. In case of high volume fractions and high aspect ratios of steel fibers as in Groups 1 and 2 and also in all concretes with hybrid high strength steel fibers, the values of equivalent flexural strength determined according to ULS are higher than those of SLSs. The reason for this is that, after the formation of the first crack, a typical strain hardening behavior is observed in these mixtures except the mixtures with low volume fractions and low aspect ratios, and also low strength of plain concrete.

Based on the test results obtained by Yalcin [15], performance classes of SFRCs for both small and large deformations (SLS and ULS), can be given in Table 3.

Tab.3. Examples of performance classes for SFRCs [18]

| Mix Code | $(f_{eq})_I$ MPa | $(f_{eq})_{II}$ MPa | Performance Classes |
|---|---|---|---|
| 80V51 | 3.54 | 4.21 | C35/45 F 3.54/4.21-XC4 |
| 65V45 | 3.11 | 2.82 | C35/45 F 3.11/2.82-XC4 |

If SFRCs are subjected to the exposure class of carbonation with wetting - drying cycle, XC4 in EN206-1 can be added to the designation shown in the last column of the above table.

## 5. Conclusions

Based on the experimental results summarized in this paper, following conclusions can be drawn:

For a certain aspect ratio, as the steel fiber volume fraction increases, mixtures with lower water cement ratio give higher equivalent flexural strengths for both serviceability and ultimate limit states.

Response surface method (RSM) is a promising approach for optimizing steel fiber reinforced concretes (SFRCs) to meet several performance criteria such as minimum cost and brittleness.

Performance classification of SFRCs can be made according to the parameters of concrete strength class and the volume fraction and aspect ratio of steel fibers.

The cost of the steel fibers used in the production of SFRCs is also important from the application point of view. Therefore, the volume fraction of steel fiber must be minimized to get an economical mixture by maximizing equivalent flexural tensile strengths. Thus, numerical optimization can optimize any combination of either factors or responses. The price of steel fibers with the high aspect ratio is higher than that of the lower ones, but their performances are contrary to their prices. Since the designer is interested in the equivalent flexural tensile strength, but not in the price, SFRC producer should find an optimum solution. Additionally, in the determination of the performance classes of SFRCs, the concrete strength, the ductility of SFRC, durability and workability in the fresh state will be of concern.

## Acknowledgements

*This research was completed in the Civil Engineering Faculty at Istanbul Technical University (ITU). The concrete production part of this study was done at Akcansa-Betonsa Technological Center. The support given by BEKAERT is gratefully acknowledged.*

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