INVERSE ANALYSIS TECHNIQUE FOR DETERMINATION OF RESIDUAL STRESS-CRACK OPENING RELATIONSHIP OF SFRC

MESKENAS Adas [1], GELAZIUS Vaidotas [2], GRIBNIAK Viktor [3], KAKLAUSKAS Gintaris [4]

Abstract

Application of steel fibre reinforced concrete (SFRC) is unceasingly increasing, but still is limited due to the absence of universally accepted and reliable design guidelines. The post-cracking load carrying capacity is the concrete property most benefited by steel fibre reinforcement, usually designed by residual strength of SFRC in tension, f_{fr}. However, quantification of f_{fr} is one of the most critical issues. Present investigation is aimed at determination of the residual stress-crack opening (σ_{fr}-w) relation of SFRC. The inverse technique was proposed for determination of σ_{fr} employing the experimental data from three-point bending tests on notched SFRC beams with different volume of fibres. Reliability of the obtained stress-crack opening σ_{fr}-w relationships and, therefore, adequacy of the proposed inverse technique was verified using nonlinear finite element analysis program ATENA. Moreover, the given σ_{fr}-w relationships were also compared with the models composed in accordance to the RILEM recommendations.

Keywords: steel fibre reinforced concrete, residual stresses, inverse technique.

1. Introduction

Steel fibre reinforced concrete (SFRC) is a cement-based material reinforced with randomly distributed fibres. The addition of fibres into the concrete matrix counteracts its brittleness, producing material with increased ductility, toughness and post-cracking stiffness. Due to great diversity of types, mechanical properties and characteristics of steel fibres, quantification of the post-cracking strength provided by interaction with concrete becomes a rather complicated issue [1]. Post-cracking strength as the main parameter, describing behaviour of SFRC, appears due to interaction between concrete and fibres crossing the crack plane. Stresses resisted by cracked SFRC are known as residual.

Generally, SFRC can be considered as a concrete with randomly dispersed fibre reinforcement or as a homogeneous material – conventional concrete with improved

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material properties. While modelling the behaviour of each discrete fibre is a complex procedure, research carried out at a macro-scale, considering SFRC as a homogeneous material, is commonly proceeded. Such analysis is often associated to the inverse analysis, which aims at determination of the parameters of a material model employing the constitutive test results [2-4].

In present work, the residual stress-crack opening (σ_{fr}-w) relation of SFRC in tension is investigated. Assuming SFRC as a homogeneous material, on the basis of the experimental data from three-point bending tests on notched members a simple and universal method for the assessment of the residual stresses is proposed. Adequacy of the obtained σ_{fr}-w relations and, therefore, the proposed inverse technique was verified using nonlinear finite element analysis program.

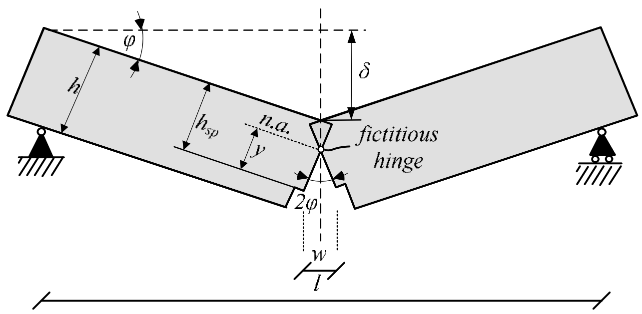

Fig. 1: Calculation scheme for the inverse analysis

2. Proposed inverse technique

The main idea of an inverse analysis is determination of the parameters of a model based on the response of structure securing the same modelling result (at the assumed tolerance) as the initially assumed one. The proposed technique aims at determination of residual stress-crack opening (σ_{fr}-w) relations using experimental measurements of the deflection and the crack tip opening displacement (CTOD) of SFRC beams.

Let us consider the cracked SFRC beam subjected to three-point loading as a mechanism of two rigid bodies interacting at a fictitious hinge (see Fig.1). It can be assumed that the location of the hinge coincides with the neutral axis position at the crack plane. According to scheme, shown in Fig.1, neglecting the elastic part of deformations and assuming that the crack surfaces remain plane (i.e. the crack opening angle equals the overall angular deformation), deflection of the element, crack opening width and position of neutral axis can be related. Geometrical relation between deflection δ and overall angular deformation φ might be given as:

$$tg\ j = 2\cdot f/l. \tag{1}$$

The distance between crack tip and neutral axis can be linked as:

$$tg\ j = w/(2\cdot y). \tag{2}$$

From the above equations, location of the neutral axis can be determined:

$$\frac{2\cdot f}{l} = \frac{w}{2\cdot y} \quad \Rightarrow y = \frac{w\cdot l}{4\cdot f}. \tag{3}$$

It should be noted that with increasing load the location of the neutral axis changes, thus it should be identified for every loading step, considered in analysis [7]. Concrete stresses in tension are neglected, as after cracking it only acts at the very near vicinity of the neutral axis and does not influence load carrying capacity [8]. Therefore, it is presumed that residual stresses are acting up to neutral axis. Inverse technique is based on assumption, that in small crack ranges w residual stresses σ_{fr} are constant, as it is shown in Fig.2.

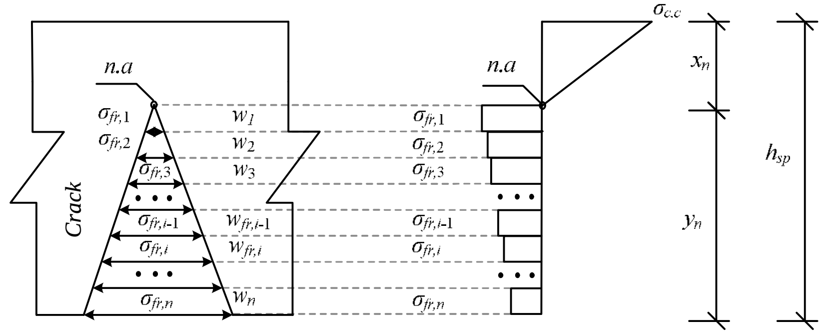

Fig. 2: Assumed residual stresses distribution

Distribution of residual stresses in a cracked section can determined as described below. Assuming small enough constant crack intervals w_i, e.g. Δw=0,05mm, the experimental load-crack opening ***P-w*** and load-deflection ***P-δ*** diagrams (Fig. 3) are divided into the parts, according to the specified crack ranges w_i (where w_i=0,05;0,10;0,15;…; w_n).

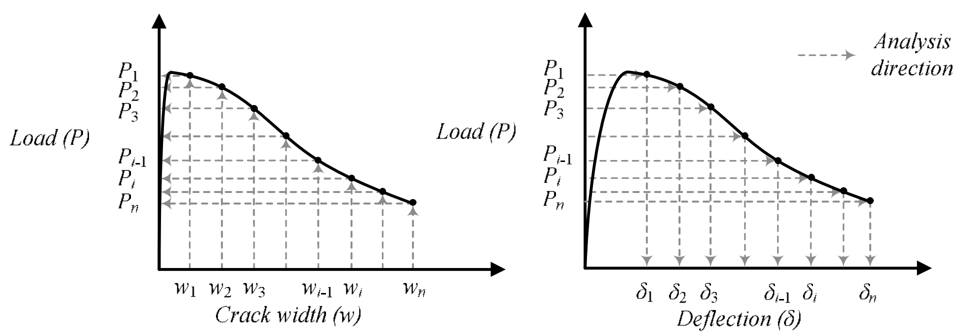

Fig. 3: Division of experimental diagrams according to the specified crack width intervals

Let us consider the ***i***-th load step. Bending moment equilibrium equation is solved with respect to the point ***O***, as shown in Fig. 4. Having residual stresses, calculated at previous loading stages ($\sigma_{fr,1}$, $\sigma_{fr,2}$, $\sigma_{fr,3}$,..., $\sigma_{fr,i-1}$), residual stresses at the load step P_i can be assessed:

$$s_{fr,i} = \frac{M_{ext,i} - \sum_{k=0}^{i-1} s_{fr,k} \cdot z_k \cdot H_i}{z_i \cdot H_i}. \tag{4}$$

Here $M_{ext,i}$ is the unit bending moment

$$M_{ext,i} = \frac{P_i \cdot l^2}{4 \cdot b}; \tag{5}$$

H_i is a height of layers with constant residual stress $\sigma_{fr,i}$ $(H_i = y_i / i)$; z_i is the distance between resultant $F_{fr,i}$ and point O of i-th crack opening (see Fig. 4)

$$z_i = \frac{y_i + 2 \cdot y_i \cdot (i-1)}{2 \cdot i} + \frac{2}{3} \cdot x_i \tag{6}$$

where y_i is the distance between the neutral axis and the notch tip; x_i is the height of compressive zone $(x_i = h_{sp} - y_i)$.

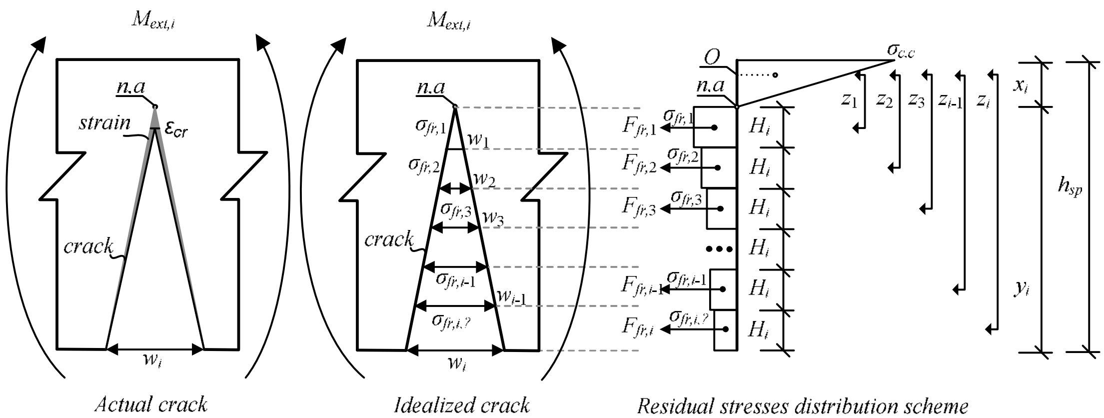

Fig. 4: Principal scheme for residual stresses analysis at ***i***-step

3. Experimental program

The experimental program consisted of six standard size beams, casted using two different concrete mixtures, containing 40 kg/m^3 and 80 kg/m^3 of steel fibres, which is respectively equivalent to V_f = 0.5% and 1.0% of the total specimen volume. Hooked end fibres applied in SFRC had the length l_f of 50 mm, a diameter d_f of 1 mm, resulting in the length-to-diameter ratio of 50. The average concrete compressive strength of 43.1 and 44.8 MPa for 0.5% and 1% of volume of concrete mixes, respectively, was obtained at 28 days with 150 mm cubes.

Three-point bending tests were carried out for determining the post-cracking behaviour of SFRC members. The test set-up was chosen according to the RILEM recommendations [6]. Standard SFRC beams with cross-sectional dimensions of 150×150 mm and length of 600 mm were tested on a span of 500 mm. To localize cracking process, each beam was notched 25 mm at the mid-span. The displacement controlled loading was applied with a rate of 0.2 mm/min.

The tests were performed under crack tip opening displacement (CTOD) control. For the measurement of CTOD, an LVDT was glued to the specimen and located at the top of the notch. In addition to the CTOD, the deflections on both sides of the beams were measured using linear variable displacement transducers (LVDT). The experimentally obtained ***P***-CTOD curves for 0.5 and 1.0% of fibre volume SFRC beams are shown in Fig. 5.

It should be noted that the test beams (made from the same concrete mixture) had demonstrated the similar post-cracking behaviour (Fig. 5). The observed scatter of results can be mainly attributed to the orientation and number of fibres crossing the crack plane.

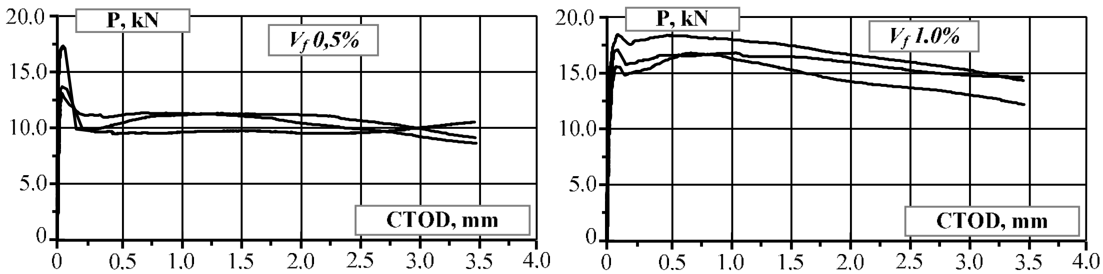

Fig. 5: Experimental load-CTOD relationships

4. Constitutive modelling results

Applying the inverse technique (Section 2) to the test measurements (Section 3), residual stresses of the cracked SFRC in tension were calculated. Obtained σ_{fr}-CTOD curves for the experimental beams with different content of fibres (0.5 and 1.0% by volume) are presented in Fig. 6.

To verify adequacy of the inverse analysis results, the obtained σ_{fr}-CTOD curves were implemented in nonlinear finite element program *ATENA* as material law of tensile concrete. Employing user-defined material model, *P*-CTOD curve was determined for the each of tested SFRC members. In Fig. 7, simulated *P*-CTOD curves are compared with the experimental data. As it can be seen, *P*-CTOD curves obtained with *ATENA* matches the experimental curves satisfactorily for both concrete mixtures.

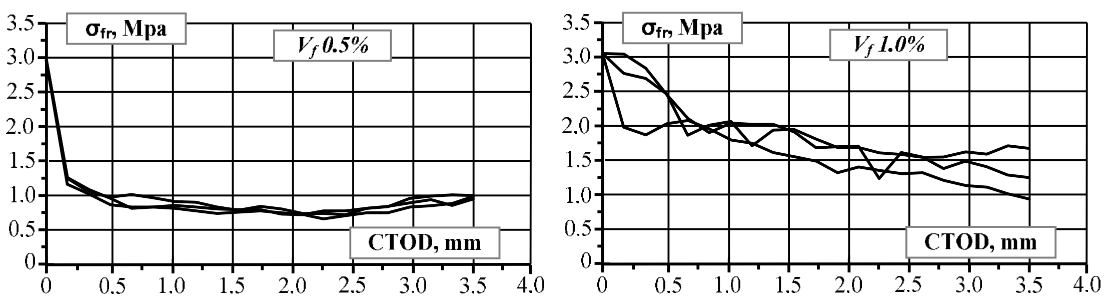

Fig. 6: Residual stress-crack opening relationships

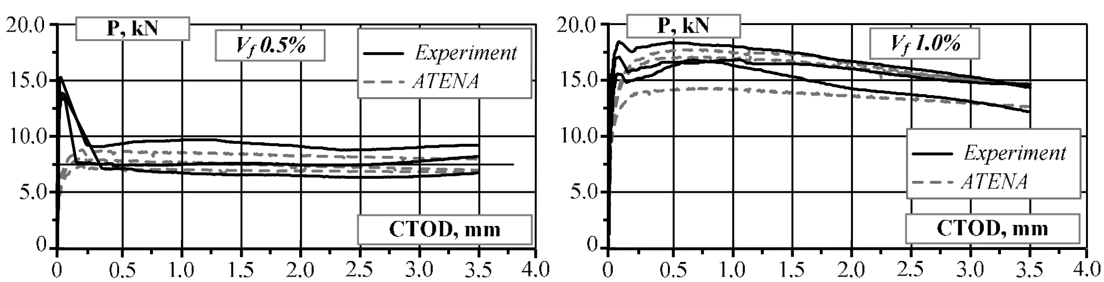

Fig. 7: Comparison of modelled and experimental results

5. Conclusions

Considering SFRC as a homogeneous material, the inverse analysis technique is proposed for determination of the residual stresses of SFRC in tension. Six standard SFRC beams tested by the authors were applied for the inverse analysis. Adequacy of the proposed method was verified using nonlinear finite element analysis program *ATENA*. The inversely derived residual stress-crack width diagrams were specified as a user defined material law for tensile concrete. The numerical simulation results indicated the ability of the inverse technique to assess accurately post-cracking behaviour of SFRC. Further research is required to extent the present investigation for different types of fibre reinforcement as well as varying geometry and sizes of the test specimens.

Acknowledgements

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