

# ANALYTICAL MODEL FOR THE GENERALIZATION OF THE BARCELONA TEST BY USING THE AXIAL DISPLACEMENT TO DETERMINE THE TOUGHNESS OF THE FRC

PUJADAS Pablo<sup>1</sup>, BLANCO Ana<sup>2</sup>, CAVALARO Sergio<sup>3</sup>,  
DE LA FUENTE Albert<sup>4</sup>, AGUADO Antonio<sup>5</sup>

## Abstract

*The need to measure the total circumferential opening displacement (TCOD) of the specimen in the Barcelona test entails the use of a circumferential extensometer, which is an expensive equipment uncommon in most control laboratories. This fact limits considerably the application of the aforementioned test. In order to simplify and to make the test more accessible, different authors have developed models to convert the axial displacement of the press ( $\delta$ ) to the TCOD, allowing the generalization of the Barcelona test by using the  $\delta$  to determine the toughness and residual strength of the FRC. In this paper a complete review, analysis and validation of those models was done comparing the experimental results with different types and dosages of fibres with the prediction obtained with the models presented.*

**Keywords:** characterization, tensile properties, Barcelona test, Double Punching test

## 1. Introduction

Most European codes and guidelines use bending tests to characterize the postcracking response of FRC and to determine the parameters defining their constitutive models. However, there are several test methods to identify, directly or indirectly, the post-cracking response of FRC. The majority of methods currently used to characterize the post-cracking behaviour of FRC [1] are based on bending tests of prismatic beams [2] loaded at mid span [3] or with two loads applied at one third of the span [4, 5]. However, several studies in the literature report a significant scatter in the results of all these test methods [6-8], frequently around 20%.

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<sup>1</sup> PUJADAS Pablo, UPC, Jordi Girona 1-3 Building C1 Room 202 08034 Barcelona (Spain), [pablo.pujads@upc.edu](mailto:pablo.pujads@upc.edu)

<sup>2</sup> BLANCO Ana, UPC, Jordi Girona 1-3 Building C1 Room 202 08034 Barcelona (Spain), [ana.blanco@upc.edu](mailto:ana.blanco@upc.edu)

<sup>3</sup> CAVALARO Sergio, UPC, Jordi Girona 1-3 Building B1 Room 106 08034 Barcelona (Spain), [sergio.pialarissi@upc.edu](mailto:sergio.pialarissi@upc.edu)

<sup>4</sup> DE LA FUENTE Albert, UPC, Jordi Girona 1-3 Building C1 Room 202 08034 Barcelona (Spain), [albert.de.la.fuente@upc.edu](mailto:albert.de.la.fuente@upc.edu)

<sup>5</sup> AGUADO Antonio, UPC, Jordi Girona 1-3 Building C1 Room 202 08034 Barcelona (Spain), [antonio.aguado@upc.edu](mailto:antonio.aguado@upc.edu)

To overcome these drawbacks, a modified double punching test known as Barcelona test was developed to measure the tensile behaviour of FRC [7] and is now a standardized method according to [8]. In fact, several researchers [7, 10-11] highlighted the advantages of this test in comparison with the bending test such as material and time saving, larger specific failure surface and, therefore, less scatter in the results, lighter specimens and the possibility of testing cores bored from real structural elements.

In the latter, two steel cylindrical punches arranged concentrically above and below the specimen transmit the load applied by the plates of the press that approach each other at a constant relative rate (Fig. 1a). This generates a radial tensile stress that causes between 2 and 4 radial cracks as well as the formation of two conical wedges at the specimen, as shown in Fig. 1b. During the test, the load applied by the press and the total circumferential opening displacement (TCOD) measured with a circumferential extensometer are recorded [9]. These results are then used to estimate FRC residual strength and toughness.

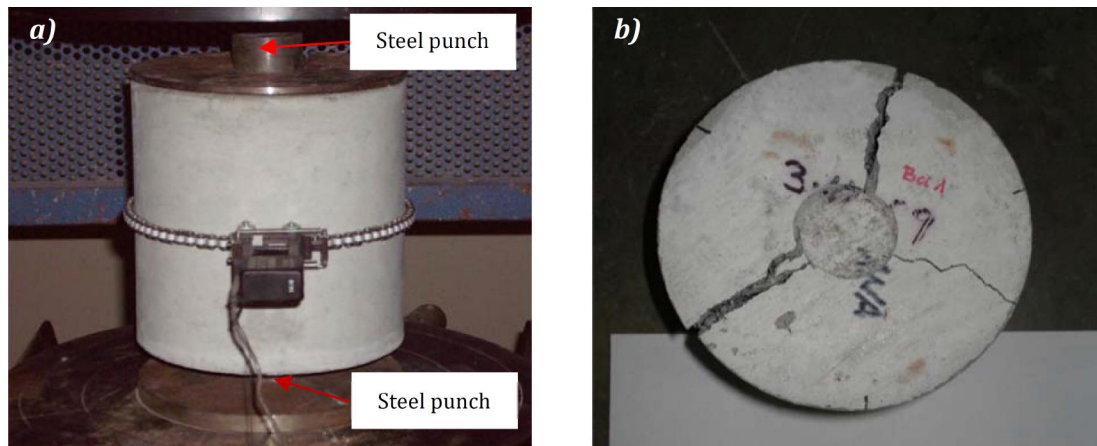


Fig. 1: a) Barcelona test setup and b) typical cracking patterns (Blanco, 2013)

Despite these advantages, the fact that a circumferential extensometer is necessary to measure the TCOD restricts the widespread use of the Barcelona test.

In order to simplify and make the test more accessible, an alternative measuring procedure was proposed by [11] to avoid using the circumferential extensometer. [11] attempted to correlate the results of toughness calculated with circumferential displacement  $E(\text{TCOD})$  and the obtained with the axial displacement  $T(\delta)$ . According to [11], a linear relation between both values exists for the range of axial displacements between 1.0 and 4.0 mm. This relation is found by means of a linear regression using experimental results. The same may be performed using the model proposed previously.

Despite the good correlation achieved by [11], the approach proposed is only valid to a restricted range of  $\delta$  between 1 mm and 4 mm. Furthermore, it does not represent the physical resistant mechanisms involved in the tensile failure of the FRC during the test. This limits the application of the correlation to the types of fibres and concrete used in the experimental program proposed by the author. In this context a more recent study by [12]

presented a more general, straightforward and accurate analytical model to convert the  $\delta$  into the TCOD for the whole extent of the curve and for any type of FRC.

Given the positive repercussion to the application of the Barcelona test and to make the characterization of the tensile behavior of FRC more accessible, this paper pursues the objective of validate the model proposed using the data from several experimental programs with different types of concrete as well as several types and amounts of fibers.

## 2. Proposals for the generalization of the Barcelona test

### 2.1 Malatesta et al. 2012

The toughness of the FRC is defined as the capacity of the material to absorb energy. In the Barcelona test, two different types of toughness may be estimated depending on the displacement used. If the axial displacement ( $\delta$ ) is considered, this property is calculated according with equation 1 that represents the area below the graph load (P) - axial displacement ( $\delta$ ). On the other hand, if the TCOD is considered, the toughness is calculated in equation 2, which gives the area below the graph load (P) – TCOD.

$$T(\delta) = \int_0^{\delta} P(\delta) d\delta \quad (1)$$

$$E(\text{TCOD}) = \int_0^{\text{TCOD}} P(\text{TCOD}) d(\text{TCOD}) \quad (2)$$

[11] attempted to correlate the results of toughness calculated with circumferential displacement  $E(\text{TCOD})$  and the obtained with the axial displacement  $T(\delta)$ . According to [16], a linear relation between both values exists for the range of axial displacements between 1.0 and 4.0 mm. This relation is found by means of a linear regression using experimental results (equation 3 for SFRC and equation 4 for PFRC):

$$E(\text{TCOD}) = 1.0410 T(\delta_A) + 82.611 \quad (3)$$

$$E(\text{TCOD}) = 1.0427 T(\delta_A) + 73.271 \quad (4)$$

### 2.2 Pujadas et al. 2013. Analytical model to convert d to TCOD

The equations developed in [12] provide a clear physical understanding of the main mechanism observed in the three Stages that the specimen undergoes during the Barcelona test. During the test, the specimen undergoes three different phases depending on its integrity and the governing resistant mechanism:

- The Stage 1 coincides with the initial application of load. The internal stress generated is resisted by the concrete matrix that presents no major cracks.
- Once the stress reaches the tensile strength of the material, the specimen enters Stage 2. The major radial cracks appear and the conical wedges are abruptly formed.

- As the cracks stabilize, the Stage 3 begins, following a kinematic mechanism that involves sliding between the conical wedge and the fragments of the cracked specimen.

A more detailed description of the specificities and the expressions governing each stage are presented in [12].

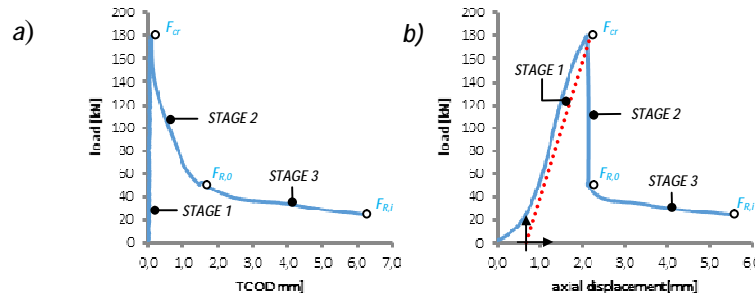


Fig. 1: Representation of a) load vs. TCOD curve, b) load vs. axial displacement ( $\delta$ ) curve and c) detail of casting imperfection in surface of specimen

Based on the aforementioned mechanisms, according to [12] the correlation between TCOD and  $\delta$  can be described by the tri-linear model presented below:

$$\begin{cases} TCOD = 0 & \delta \leq \delta_{cr} \\ TCOD = n \cdot \frac{\alpha \cdot \delta_{R,0}}{2 \cdot l} \cdot \sin \frac{\pi}{n} \left( 1 - \frac{F}{F_{cr}} \right) & \delta_{cr} < \delta < \delta_{R,0} \\ TCOD = n \cdot \frac{\alpha}{2 \cdot l} \cdot \sin \frac{\pi}{n} \cdot \left[ \delta - \delta_{cr} + \delta_{R,0} \cdot \left( 1 - \frac{F_{R,0}}{F_{cr}} \right) \right] & \delta \geq \delta_{R,0} \end{cases} \quad (5)$$

The application of the model proposed in [12] is direct and valid for the whole extent of the load-axial displacement ( $\delta$ ) curve. It depends only on the definition of 6 physical input parameters:  $F_{cr}$ ,  $\delta_{cr}$ ,  $F_{R,0}$ ,  $\delta_{R,0}$ ,  $n$ , and  $l$ . All these parameters are either obtained during the Barcelona test or assessed afterwards upon a visual inspection of the specimen. Then, the equations defined previously are used to convert each value of axial displacement ( $\delta$ ) to a corresponding TCOD. This allows the definition of the resulting load-TCOD curve.

### 3. Validation of the experimental results

#### 3.1 Validation of Pujadas et al. 2013

A wide range of FRC in terms of strength (with softening and hardening), fiber type and fiber content was considered. Results from 3 concrete mixes were used: 1 FRC with polypropylene fibres (PF), 1 FRC with steel fibres (SF) and 1 Ultra High Fiber Reinforced Concrete (UHPC) with steel microfibres (SMF).

The fibre content added to the mix and the nomenclature adopted to distinguish between these concretes are presented in Table 1. The main characteristics of the fibres used are presented in Table 2.

Tab.1: Types of concrete

Nomenclature	Type of concrete	Fiber type	Fiber content
FRC_PF_9_N	FRC	PF	9 kg/m <sup>3</sup>
FRC_SF_40_N	FRC	SF1	40 kg/m <sup>3</sup>
UHPRFC_SMF_130_N	UHPRFC	SMF	130 kg/m <sup>3</sup>

Tab.2: Fiber characteristics (data provided by the manufacturer).

Characteristics	Polypropylene Fiber		Steel Fiber
	PF	SF	MSF
Length (mm)	48mm	50	12
Equivalent diameter (mm)	-	0,62	0,20
Aspect ratio	-	80	60
Tensile strength (MPa)	550	1270	1270
Modulus of elasticity (GPa)	6.0	210	210
Number of fibers per kg	>35.000	8.100	-

The Barcelona test was performed in all specimens following the [9]. Both the TCOD and axial displacement ( $\delta$ ) of the press were measured during the test. According to the validation procedure defined, initially the parameters required for the application of the model proposed in this paper are identified (see Table 3). The values of  $F_{cr}$ ,  $\delta_{cr}$ ,  $F_{R,0}$  and  $\delta_{R,0}$ , were estimated directly in the curve load-axial displacement of each specimen. A length of the conical wedge ( $l$ ) of 40 mm was used for all specimens since the direct measurement could not be performed. The number of cracks was assessed visually in some cases and the reference value of 3 was assumed in specimens without any information regarding this number.

Tab.3: Input parameters used to predict the TCOD in the Barcelona Test

Specimen	$F_{fis}$ [kN]	$\delta_{fis}$ [mm]	$F_{res}$ [kN]	$\delta_{res}$ [mm]	$n$ [-]	$l$ [mm]
FRC_PF_9_1	152.60	1.36	96.00	1.43	4	40
FRC_PF_9_2	149.50	1.70	84.50	1.82	3	40
FRC_PF_9_3	140.95	1.57	81.90	1.59	3	40
FRC_PF_9_4	151.90	1.76	88.00	1.79	4	40
FRC_PF_9_5	155.30	1.85	87.79	1.85	3	40
FRC_PF_9_6	155.30	2.36	71.71	2.39	3	40
FRC_SF_40_1	150.50	1.23	112.50	1.26	3*	40
FRC_SF_40_3	157.70	1.33	116.00	1.45	3*	40
FRC_SF_40_4	157.70	1.13	119.00	1.19	3*	40
FRC_SF_40_5	158.37	1.37	110.00	1.39	3*	40
FRC_SF_40_6	151.40	1.33	105.00	1.36	3*	40
UHPRFC_SMF_130_1	323.00	1.06	323.00	1.06	3*	40
UHPRFC_SMF_130_2	325.00	1.16	325.00	1.16	3*	40

The data provided in Table 3 is used in the model defined in section 4 to convert the axial displacement ( $\delta$ ) to the TCOD. Finally, the latter is compared with the TCOD measured during the test of each specimen.

Fig. 2 to 4 illustrates the results of the application of the model proposed in one specimen representative of each type of concrete tested. In every figure, the first graph shows the curve of the load applied and the axial displacement measured ( $\delta$ ) during the test. The second graph presents the curve relating the axial displacement ( $\delta$ ) and the TCOD obtained in the test and estimated with the model proposed in section 2.2.

Likewise, the third graph compares the final TCOD-load curve obtained experimentally and the one estimated theoretically from the axial displacement ( $\delta$ ).

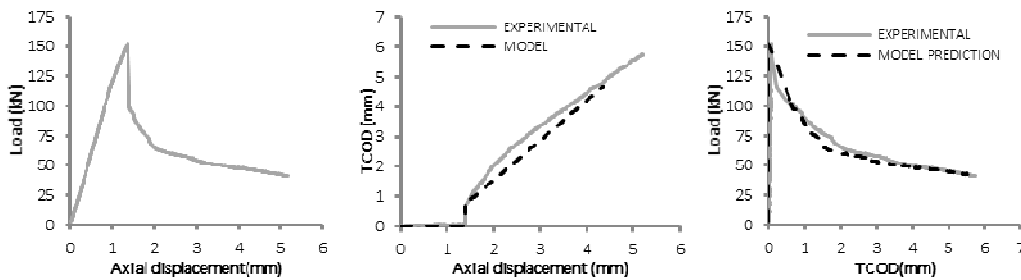


Fig. 2: Results for FRC\_PF\_9\_1.

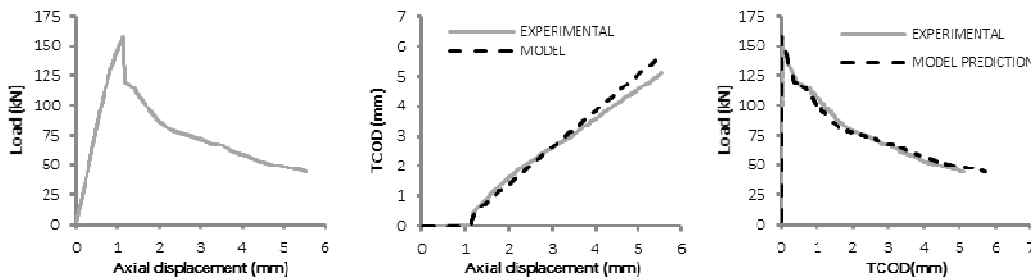


Fig. 3: Results for FRC\_SF\_40\_4.

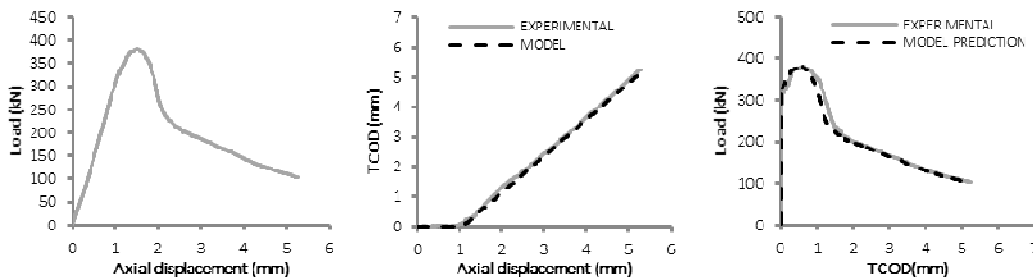


Fig. 4: Results for UHPFRC\_SMF\_130\_1.

### 3.2 Comparison between both models

Table 4 presents the predicted values of  $E(TCOD < 4mm)$  for each of the specimens tested with both Carmona's and the new model proposed, together with the experimental results and the relative error of the prediction.

Tab.4: Comparison of experimental and predicted values of  $E(TCOD < 4mm)$ .

Specimen	$E(TCOD < 4mm)$ [kN·mm]				
	EXP.	Carmona et al.		New Model	
		PRED.	ERR. (%)	PRED.	ERR. (%)
FRC_PF_9_1	316.04	357.80	-13.21	300.97	4.77
FRC_PF_9_2	244.68	357.26	-46.01	283.65	-15.93
FRC_PF_9_3	273.51	353.93	-29.40	273.19	0.12
FRC_PF_9_4	281.29	362.19	-28.76	306.47	-8.95
FRC_PF_9_5	232.87	392.03	-68.35	296.81	-27.46
FRC_PF_9_6	246.44	291.16	-18.15	285.41	-15.81
FRC_SF_40_1	308.22	407.38	-32.17	302.25	1.94
FRC_SF_40_3	304.17	406.07	-33.50	309.15	-1.64
FRC_SF_40_4	318.95	429.98	-34.81	333.52	-4.57
FRC_SF_40_5	316.76	407.71	-28.71	306.52	3.23
FRC_SF_40_6	304.96	372.37	-22.10	274.36	10.03
UHPRFC_SMF_130_1	876.03	1315.73	-50.19	867.69	0.95
UHPRFC_SMF_130_2	469.65	1086.90	-131.43	583.36	-24.21

There is a good agreement between the experimental results and the predictions performed with the new model of [12]. In some cases, however, a high relative error higher than 10% may be found. These high values could be the product of the definition of a constant length of the conical wedge ( $l = 40 \text{ mm}$ ) or number of cracks ( $n = 3$ ), which were fixed in order to avoid a biased analysis given the lack of detailed information.

It is reasonable to consider that an even better approximation of the experimental results could be obtained if more data about the specimens tested was available.

The values obtained with the model from [11] clearly overestimate (in all cases) the  $E(TCOD < 4mm)$ , leading to relative errors considerably higher than the obtained with the new model (see Figure 5).

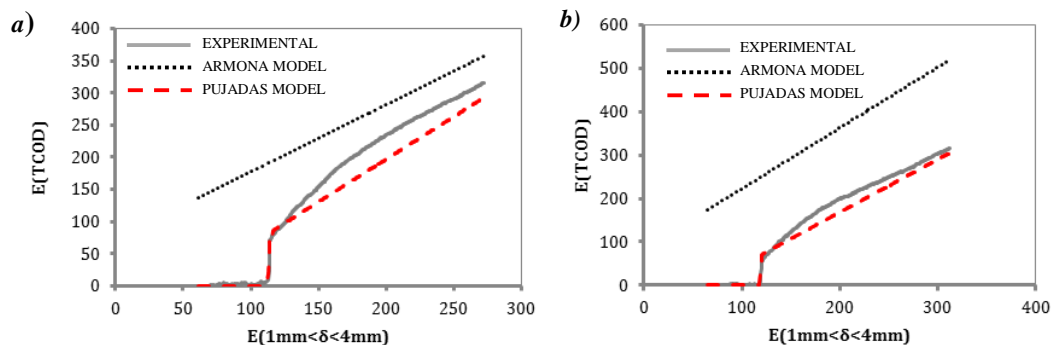


Fig. 5: Relación entre  $E(0) - E(TCOD)$  a) HRF\_PF\_9\_1 b) HRF\_SF\_40\_5

## 4. Conclusions

The Barcelona Test has proved to be suitable for the systematic control of the tensile properties of FRC.

Nevertheless, the need to measure the total circumferential opening displacement (TCOD) of the specimen entails the use of an expensive circumferential extensometer, which limits the application of the test. Different models to generalize this test were developed.

The main conclusions derived from the study performed in this paper are listed below:

- The new model developed in [12] provided a clear physical understanding of the main mechanism observed in the three stages that the specimen undergoes during the Barcelona test.
- This model considers parameters like the length of the cone ( $l$ ) and the number of cracks ( $n$ ) that may be measured after the test for each specimen.
- The comparison with the experimental result indicate that the model is capable of predicting accurately the entire curve, regardless of the type of concrete (conventional FRC or UHPFRC), of the type of fibre (steel or plastic, micro or macro) and of the post-cracking behaviour (with hardening or with softening).
- A good agreement between the measured and the predicted values was found for all cases. The average error of the estimation of the toughness with the experimental equation from [11] yields a higher average error than [12] for the same group of specimens.
- The results obtained in this chapter validate the new model proposed, thus allowing a positive simplification of the Barcelona test that can be performed with control of axial displacement instead of with control of TCOD.

The development of this model allows at the same time the characterization of PFRC by means of samples with a shape different from the cylindrical, such as the cubic specimen used in the multidirectional punching test (MDPT) [13].

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## References

- [1] Montaignac, R., Massicotte, B., Charron, J.P., and Nour, A. *Design of SFRC structural elements: post-cracking tensile strength measurement. Materials and Structures*, Vol 45 (4), pp. 609-622
- [2] Molins, C., Aguado, A. y Marí, A. (2006). *Quality control test for SFRC to be used in precast segments. Tunnelling and underground space technology*, Vol. 21.
- [3] EN 14651 (2005) *Test method for metallic fibered concrete – Measuring the flexural tensile strength (limit of proportionality (LOP), residual)*, European Committee for Standardization, Brussels.
- [4] NBN B 15-238 (1992) *Test on fibre reinforced concrete bending test on prismatic simples. Norme Belge, Institut Belge de Normalisation, Brussels (in French).*
- [5] ASTM C-1018 (1997) *Standard test method for flexural toughness and first-crack strength of fiber-reinforced concrete (using beam with third-point loading)*. American Society for Testing and Materials, Philadelphia
- [6] Dupont, D. (2003) *Modelling and experimental validation of the constitutive law ( $\sigma$ - $\epsilon$ ) and cracking behavior of steel fibre reinforced concrete*, PhD Thesis, Katholieke Universiteit Leuven.
- [7] Molins, C., Aguado, A. and Saludes, S. (2008) *Double Punch Test to control the tensile properties of FRC (Barcelona test)*. *Materials and Structures (RILEM)*. Vol. 42, nº 4. May 2009 pp.: 415-425. ISSN: 1359-5997
- [8] Parmentier, B., Vandewalle, L. and Van Rickstal, F. *Dispersion of the mechanical properties of FRC investigated by different bending tests*, *Tailor Made Concrete Structures*, Walraven and Stoelhost, eds, Taylor & Francis Group, 2008, pp. 507-512.
- [9] UNE 83515:2010 (2010) *Hormigones con fibras. Determinación de la resistencia a fisuración, tenacidad y resistencia residual a tracción. Método Barcelona. AENOR*
- [10] Chao, S.-H., Karki, N.B., Cho, J.-S. and Waweru, R.N. *Use of double punch test to evaluate the mechanical performance of fiber reinforced concrete*. In *proceedings of the 6th International Workshop on High Performance Fiber Reinforced Cement Composites (HPFRCC6)*, Ann Arbor, Michigan, June 20-22 2011, Parra-Montesinos, ReinhardtNaaman Eds., 26-35.
- [11] Carmona, S.; Aguado, A. y Molins, C. (2012) *Generalization of the Barcelona test for the toughness control of FRC*. *Materials and Structures*, 45(7), pp. 1053-1069.
- [12] Pujadas, P.; Blanco, A.; Cavalaro, S.; de la Fuente, A. y Aguado, A. (2013) *New analytical model to generalize the Barcelona test using axial displacement*. *Journal of Civil Engineering and Management*. Vol. 19(2), pp. 259-271.
- [13] Pujadas, P. (2013) *Caracterización y diseño del hormigón reforzado con fibras plásticas*. Doctoral Thesis, Universitat Politècnica de Catalunya, Barcelona.