STRESS-STRAIN STATE ANALYSIS OF REINFORCED CONCRETE BEAMS WITH STEEL FIBERS

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Abstract

Paper presents and analyzes the experimental program results which consider influence of steel fiber addition on the stress-strain state of reinforced concrete beams up to fracture during shot-term ultimate static load. Concrete beams were made of four types of concrete: ordinary concrete (OSC) – C30/37, ordinary concrete with steel fibers (FROSC) – C30/37, high strength concrete (HSC) – C60/75, high strength concrete with steel fibers (FRHSC) – C60/75. Addition of steel fibers was in low percentage of 0.45%. Apart from influence of fiber addition, the effect of concrete age was analyzed. The ratio of load and deflection in the center of the beam (U2) is defined at shot term ultimate load up to fracture with one unloading cycle (from the level of the designed exploitation load to the level of permanent load). Dilatation in concrete and reinforcement by cross section beam height were also analyzed. Results indicate that there is high significance of fiber addition on beams deflection particularly for HSC, but there is a small influence on ultimate load increase, for both OSC and HSC.

Keywords: steel fibres; ordinary strength concrete; high-strength concrete; short-term loads

1. Introduction

Concrete reinforced with steel fibers is a composite material the properties of which differ from those of concrete and steel fibers when taken separately. Concrete properties are mainly changed by steel fibers: strength and strain at tension, flexion and elasticity modulus are increased and other mechanical properties are enhanced.

It is well known that when exposed to bending fiber reinforced concrete section with steel fibers, after cracking, redistribution of stress occurs, where the neutral axis shifts in the pressured zone direction and reaching a new equilibrium state. As a result of this effect, sections of fiber reinforced concrete are able to absorb greater moments than ordinary concrete i.e. they have a higher bearing capacity. It follows that observing relationship diagram momentum (stress) - deflection (strain) after reaching a proportionality limit i.e.

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“strength before the first crack”, fiber reinforced concrete has a much higher dilatation than ordinary concrete [1].

This fact was confirmed within numerous experimental programs that have been worked out to identify the flexural behavior of steel fiber reinforced concrete (SFRC) beams [2]; [3].

It was found that SFRC beams exhibit higher flexural strength, ductility and toughness properties for higher fiber content [4]; [5]; [6].

This paper presents the results of a wider experimental studies carried out on 16 reinforced concrete beams with cross section 15x28 and length 300 cm, and 120 test body’s 15x15x30 cm for determining the mechanical properties of concrete. The beams were made of four types of concrete. Concrete without fibres were class C30/37 (OSC – ordinary strength concrete) and C60/75 (HSC – high strength concrete), and with steel fibres corresponding types of concrete labelled as FROSC and FRHSC. Fibres used were type IRI 50/30 (VJ2) with tensile strength of 900 MPa in an amount of 0.45% of the volume. Beams were subjected to bending test with two concentrated forces. The research results are related to stress and strain state and dilatation-sectional height, under short-term ultimate load up to fracture with one unloading cycle (from the level of the designed exploitation load to the level of permanent load) from two aspects: the effect of steel fiber and the effect of concrete age (for the period t=35-40 days and t=420-430 days).

2. Experimental Research

2.1 Experimental program

Four types of concrete were used. Mix proportioning of the concrete for the classes C30/37 (OSC) and C60/75 (HSC) are given in table 1.

Test beams were divided in four series A, F, G and H (table 2). The beams were reinforced in tensile zone by deformed bars 2ϕ12, in compression zone by deformed bars 2ϕ8, class B400 and stirrups ϕ6, class 240.

A and G series beams were tested on short-term ultimate load after t=35–40 days from the day of concreting, series F and H, t=420–430 days from the day of concreting. Until the moment of loading, all elements (beams), as well as the test specimens were treated completely the same way.

The experimental tests were carried out at Faculty of Civil Engineering in Skopje (Macedonia), in ambient of approximately constant average humidity values RH_m=63% and temperature T_m=17ºC, in the test period t=430 days.

Configuration of measuring points (figure 1) was determined with the goal to have a better insight in the behaviour of beams throughout time, as well as with the effect of short-term ultimate load. The labels of specific measuring points are as follows:

- measuring points for measurement of dilatations in reinforced – concrete beams by using the deformeters are labelled as D on the beams;
- measuring points for measuring dilatations by using measuring tapes on the concrete are labelled as B, and on reinforcement as A;

- measuring points for measuring flexion, deformations of deflection meter are labelled as U in the medium beam zone and with O on beam support.

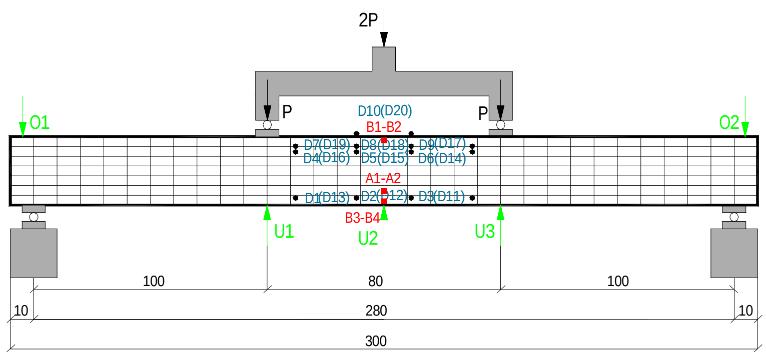

Fig. 1: Configuration of measuring spots on test elements (longitudinal section)

Tab.1: Mix proportioning of concrete

Materials	**Type**	**Concrete class C30/37 (OSC) [kg/m^3]**	**Concrete class C60/75 (HSC) [kg/m^3]**
Cement	CEM I 42.5R	360	414
Mineral admixtures	Silica fume 8%	/	36
Water	well	210	158
W/C ratio		0.58	0.36
Chemical admixtures	superplasticizer 1.1%	/	1.1%
Aggregate:		1850	1870
I (0-4)mm	River sand	40%	37%
II (4-8)mm	Crushed limestone	22%	25%
III (8-16)mm	Crushed limestone	38%	38%

Tab.2: Test beams per series

Series	**Group**	**Concrete type**	**Series**	**Group**	**Concrete type**	**Test time [days]**
A	A_1	OSC	G	G_1	FROSC	t=35-40
	A_2	OSC		G_2	FROSC	t=35-40
	A_3	HSC		G_3	FRHSC	t=35-40
	A_4	HSC		G_4	FRHSC	t=35-40
F	F_1	OSC	H	H_1	FROSC	t=420-430
	F_2	OSC		H_2	FROSC	t=420-430
	F_3	HSC		H_3	FRHSC	t=420-430
	F_4	HSC		H_4	FRHSC	t=420-430

2.2 Results and analysis of stress-strain beam state with short-term ultimate load effect

Within this analysis, we examined stress-strain state with short-term ultimate load effect with all 16 test elements (beams) according to following effects: the effect of fibres addition and concrete age (t=35-40 days, t=420-430 days). We examined the relation between load and deflection in the middle of the beam (U2) as well as the relation of dilatations in concrete and reinforcement according to height of beam cross-section, with one unloading cycle: the level of permanent load (g) 2P=8 kN, the level of exploitation load (g+p) 2P=23,2 kN up to fracture.

2.2.1 Load-strains (deflection)

For determining the **effect of steel fibres** addition on the process and characteristic strain values (deflection) with short-term ultimate load effect, following diagrams illustrate the load-deflection relation in the middle of the beam, for two different beam ages t=35-40 days and t=420-430 days (figures 2 and 3).

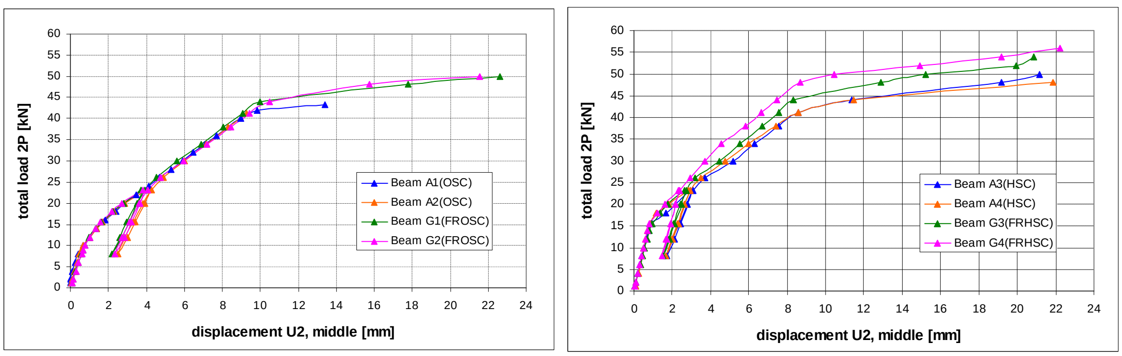

Fig. 2: Force-strain (deflection) relation for beams of A and G series

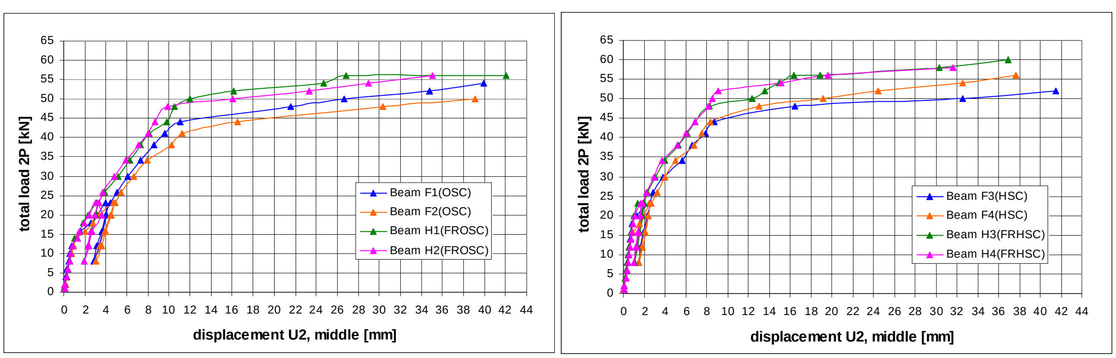

Fig. 3: Force-strain (deflection) relation for beams of F and H series

Tables 3 and 4 gives values and relations of ultimate load F_u, characteristic strain values (on the level of permanent load Δ_{Fg}, exploitation load Δ_{Fg+Fp}, permanent unload $\Delta_{Fg,unload}$) beams of series A, G and beams of series F, H, according to determined relation of concrete mixtures without and with 0,45% of steel fibres.

Tab.3: Strain relation for beams A and G series

Series	F_u [kN]	Δ_{Fg} [mm]	Δ_{Fg+Fp} [mm]	$\Delta_{Fg,unload}$ [mm]	$\Delta_{Fg,unload}$/ Δ_{Fg}
A1,2,$_{av.}$	56,10	0,44	4,11	2,48	5,64
G1,2,$_{av.}$	57,63	0,57	3,79	2,26	3,96
G1,2,$_{av.}$/A1,2,$_{av.}$	1,03	1,30	-1,08	-1,10	-1,42
A3,4,$_{av.}$	62,58	0,44	2,89	1,71	3,93
G3,4,$_{av.}$	62,43	0,43	2,24	1,25	2,91
G3,4,$_{av.}$/A3,4,$_{av.}$	1,00	-1,01	-1,29	-1,37	-1,35

Tab.4: Strain relation for beams F and H series

Series	F_u [kN]	Δ_{Fg} [mm]	Δ_{Fg+Fp} [mm]	$\Delta_{Fg,unload}$ [mm]	$\Delta_{Fg,unload}$/ Δ_{Fg}
F1,2,$_{av.}$	55,55	0,51	4,33	2,94	5,81
H1,2,$_{av.}$	59,73	0,52	3,11	1,96	3,80
H1,2,$_{av.}$/F1,2,$_{av.}$	1,08	1,02	-1,39	-1,50	-1,53
F3,4,$_{av.}$	64,78	0,39	2,48	1,50	3,90
H3,4,$_{av.}$	68,25	0,42	1,81	1,08	2,59
H3,4,$_{av.}$/F3,4,$_{av.}$	1,05	1,08	-1,37	-1,40	-1,50

In order to determine **effect of concrete age** on the course and characteristic strain (deflection) values with short-term ultimate load effect, following diagrams illustrate the load – deflection relation in the middle of the beam for two concrete age t=35-40 days and t=420-430 days for all four concrete mixtures (figures 4 and 5).

Tables 5 and 6 illustrate values and relation of ultimate load F_u, characteristic strain values with beams of A series and G series, tested with t=35-40 days and beams of F series and H series tested with t=420-430 days.

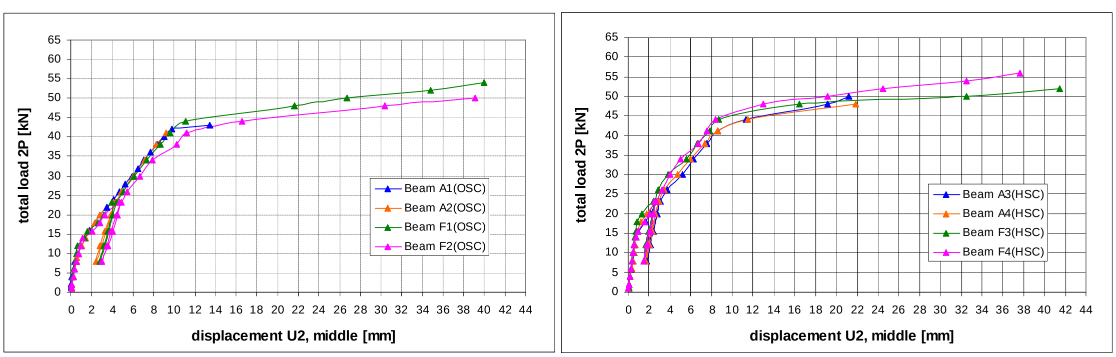

Fig. 4: Force-strain (deflection) relation for beams of A series and F series

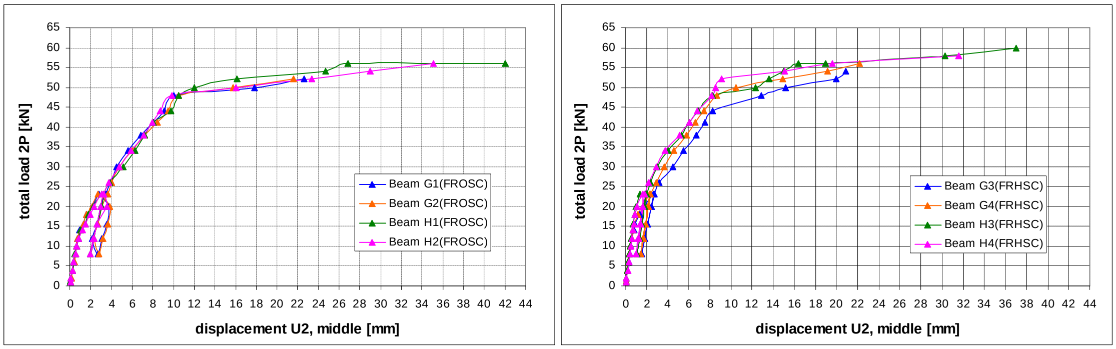

Fig. 5: Force-strain (deflection) relation for beams of G series and H series

Tab.5: Strain relation for beams A and F series

Series	F_u [kN]	Δ_{Fg} [mm]	Δ_{Fg+Fp} [mm]	$\Delta_{Fg,unload}$ [mm]	$\Delta_{Fg,unload}/\Delta_{Fg}$
A1,2,$_{av.}$	56,10	0,44	4,11	2,48	5,64
F1,2,$_{av.}$	55,55	0,51	4,33	2,94	5,81
F1,2,$_{av.}$/A1,2,$_{av.}$	-1,01	1,15	1,05	1,18	1,03
A3,4,$_{av.}$	62,58	0,44	2,89	1,71	3,93
F3,4,$_{av.}$	64,78	0,39	2,48	1,50	3,90
F3,4,$_{av.}$/A3,4,$_{av.}$	1,04	-1,13	-1,17	-1,14	-1,01

Tab.6: Strain relation for beams G and H series

Series	F_u [kN]	Δ_{Fg} [mm]	Δ_{Fg+Fp} [mm]	$\Delta_{Fg,unload}$ [mm]	$\Delta_{Fg,unload}/\Delta_{Fg}$
G1,2,$_{av.}$	57,63	0,57	3,79	2,26	3,96
H1,2,$_{av.}$	59,73	0,52	3,11	1,96	3,80
H1,2,$_{av.}$/G1,2,$_{av.}$	1,04	-1,11	-1,22	-1,16	-1,04
G3,4,$_{av.}$	62,43	0,43	2,24	1,25	2,91
H3,4,$_{av.}$	68,25	0,42	1,81	1,08	2,59
H3,4,$_{av.}$/G3,4,$_{av.}$	1,09	-1,04	-1,24	-1,16	-1,12

3. Dilatations according to height of the section with short-term ultimate load effect

With all 16 test elements (beams) we measured dilatations according to the height of beam cross-section (h=28 cm) with short-term ultimate load effect, according to following effects, type of concrete mixtures (OSC, HSC, FROSC, FRHSC), i.e. effect of concrete strength and fibre addition, and concrete age effect (t=35-40 days, t=420-430 days).

With deformeter we monitored the dilatations on 20 measuring spots and 6 measuring tapes (figure 6), per nine measuring spots on side ends and two measuring spots on the beam top, while per two measuring tapes were on the concrete surface (on the bottom of beams B3, B4 and on the top of beams B1, B2) and two on tensile reinforcement (A1, A2).

We gave configuration of dilatations according to beam height for each beam (example on figure 7) according to short-term load effect, for three load levels: the level of permanent load (g) 2P=8 kN, the level of exploitation load (g+p) 2P=23,2 kN and load level up to which it was possible to measure dilatation on the level of tensile reinforcement (D2-D12) (reinforcement dilatations above the limit). This third level of load was moving in the small scope of load intensity of 2P=48 kN for beams without steel fibres to 2P=52 - 54 kN for beams with steel fibres, so that we can compare obtained results.

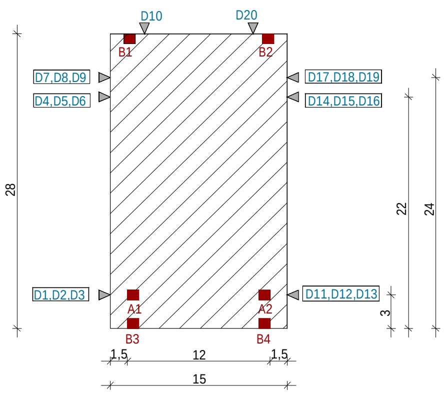

Fig. 6: Configuration o measuring spots on AB beams (cross-section)

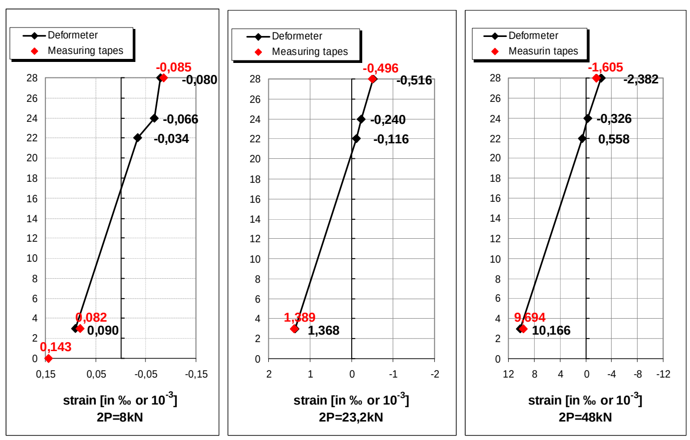

Fig. 7: Example of measured dilatations in concrete and reinforcement according to the height of beam, sample A1

Within this analysis we examined the **effect of steel fibres** (0,45% compared to total concrete volume) on final dilatations values measured in concrete and reinforcement, that is on compressed concrete edge and on the tensile rods level on short-term tensile bending.

Tab.7: Ratio of maximally measured dilatations for beams A and G

Series A/G	ε_a [‰]	ε_b [‰]	F_a [kN]	F_b [kN]	F_{max} [kN]
$A1,2_{av.}$	9,197	-2,864	49	51	56,10
$G1,2_{av.}$	8,456	-3,350	49	52	57,625
$G1,2_{av.}/A1,2_{av.}$	-1,09	1,17	1,00	1,02	1,03
$A3,4_{av.}$	8,289	-1,966	48	52	62,575
$G3,4_{av.}$	9,136	-2,094	54	55	62,425
$G3,4_{av.}/A3,4_{av.}$	1,10	1,07	1,13	1,06	1,00

Tab.8: Ratio of maximally measured dilatations for beams F and H

Series F/H	ε_a [‰]	ε_b [‰]	F_a [kN]	F_b [kN]	F_{max} [kN]
F1,2,av.	8,738	-3,570	46	53	55,55
H1,2,av.	9,385	-3,962	52	56	59,725
$H1,2_{av.}/F1,2_{av.}$	1,07	1,11	1,13	1,06	1,08
F3,4,av.	7,999	-2,963	49	60	64,775
H3,4,av.	9,309	-2,444	54	60	68,25
$H3,4_{av.}/F3,4_{av.}$	1,16	-1,21	1,10	1,00	1,05

Ratio of maximally measured dilatations on compressed concrete edge ε_b, on the level of tensile rods ε_a, with ratio of force F_b with which were measured maximal dilatations on the compressed section edge, force F_a with which were measured maximal dilatations on the level of tensile rods and ratio of ultimate load F_{max}, is given in tables 7 and 8.

For determining of **effect of concrete age** on ultimate measured dilatation values in concrete and reinforcement, i.e. on compressed concrete edge and on the level of tensile rods of beams stressed on short-term ultimate bending load, the following tables (tables 9 and 10) illustrate the ratio of maximally measured dilatations ε_b and ε_a, ratio of force F_b, force F_a and the ratio of ultimate load F_{max}, for two beam ages t=35-40 days and t=420-430 days, all four concrete mixtures.

Tab.9: Ratio of maximally measured dilatations for beams A and F

Series A/F	ε_a [‰]	ε_b [‰]	F_a [kN]	F_b [kN]	F_{max} [kN]
$A1,2_{av.}$	9,197	-2,864	49	51	56,10
$F1,2_{av.}$	8,738	-3,570	46	53	55,55
$F1,2_{av.}/A1,2_{av.}$	-1,05	1,25	-1,07	1,04	-1,01
$A3,4_{av.}$	8,289	-1,966	48	52	62,575
$F3,4_{av.}$	8,744	-2,963	49	60	64,775
$F3,4_{av.}/A3,4_{av.}$	1,05	1,51	1,02	1,15	1,04

Tab.10: Ratio of maximally measured dilatations for beams G and H

Series G/H	ε_a [‰]	ε_b [‰]	F_a [kN]	F_b [kN]	F_{max} [kN]
$G1,2_{av.}$	8,456	-3,350	49	52	57,625
$H1,2_{av.}$	9,385	-3,962	52	56	59,725
$H1,2_{av.}/G1,2_{av.}$	1,11	1,18	1,06	1,08	1,04
$G3,4_{av.}$	9,136	-2,094	54	55	62,425
$H3,4_{av.}$	9,309	-2,444	54	60	68,25
$H3,4_{av.}/G3,4_{av.}$	1,02	1,17	1,00	1,09	1,09

Table 11 gives the ultimate dilatations ε_b and ε_a, force F_b, force F_a and ultimate load F_{max} (force registered on dynamo meter in the moment of beam fracture). Fracture represents state of breaking of bigger and/or smaller concrete parts or state of reaching bigger cracks width.

Tab.11: Maximally measured dilatations with short-term ultimate load effect

Series	Groups	Concrete type	ε_a [‰]	ε_b [‰]	F_a [kN]	F_b [kN]	F_{max} [kN]
A	A1	OSC	10,166	-2,766	48	50	55,40
	A2	OSC	8,228	-2,962	50	52	56,80
	A3	HSC	8,020	-1,516	48	50	62,65
	A4	HSC	8,558	-2,416	48	54	62,50
F	F1	OSC	8,622	-3,716	48	54	56,00
	F2	OSC	8,854	-3,424	44	52	55,10
	F3	HSC	8,676	-3,222	48	60	63,80
	F4	HSC	8,812	-2,704	50	60	65,75
G	G1	FROSC	8,480	-3,496	48	52	57,70
	G2	FROSC	8,432	-3,204	50	52	57,55
	G3	FRHSC	9,818	-2,128	52	54	61,15
	G4	FRHSC	8,454	-2,060	56	56	63,70
H	H1	FROSC	8,542	-4,044	52	56	59,70
	H2	FROSC	10,228	-3,880	52	56	59,75
	H3	FRHSC	9,358	-2,394	54	60	67,20
	H4	FRHSC	9,260	-2,494	54	60	69,30

Maximally measured dilatations values on compressed concrete edge with short-term load effect are in a range from -1,516‰ to -4,044‰. The highest values of dilatations were registered with series H beams with addition of steel fibres and that are tested after t=420-430 days and the lowest with series A beams without steel fibres additive, tested on short-term ultimate load after t=35-40 days. Dilatations on the level of tensile rods are quite monotonous and they are in range from 8,020‰ to 10,228‰. Ultimate loads are being increased with fibrous beams even in the case of ordinary concrete and high strength concrete beams.

4. Conclusions

Based on the experimental results analyses we can draw following conclusions:

- Addition of a low percentage of 0.45% fibers to concrete is causing an increase of ultimate load F_u to 3% for the OSC, but there is no significant effect for the HSC, with relatively small concrete age of 35-40 days. At higher age (420-430 days), the percentage for OSC increases to 8% and 5% for HSC.
- Significant reduction of deflection can be registered with both types of concrete after the addition of fibers and this ratio goes up to 50%, which means that adding fibers lowers the deflection.
- The effect of concrete age for mixtures with steel fibers is reflected in a slightly higher ultimate force of 4% for FROSC and 9% for FRHSC at concrete age of 420-

430 days. Also, greater deformation (deflection) reduction for older concrete was observed and it was more expressive for high strength concrete (about 12%).

- Dilatation of reinforcement and concrete do not have a uniform increase or decrease and results vary considerably. Addition of steel fibers to concrete age 35-40 days is reflected in the reduction of reinforcement dilatation of 9% for ordinary strength concrete, while for high-strength concrete, there is an increase of 10%. Dilatations of concrete are by 17% higher for FROSC and only 7% for FRHSC. For concrete age 420-430 days an increase in reinforcement dilatation of 7% for ordinary concrete and 16% for high strength concrete is registered. Dilatations of concrete are opposite signs and manifest in the increase of 11% for ordinary concrete, and a decrease of 21% for high-strength concrete.
- From the aspect of concrete age fiber addition affects the increase of dilatation in steel reinforcement by 11% for ordinary concrete and only 2% for high-strength concrete. Dilatations of concrete in both cases are very similar and amount to 18 i.e. 17%.

Presented results are given for concrete elements which correspond in size and percentage of reinforcement to elements used in practice. The results for timeframe of 420-430 days are especially significant, because there are a small number of experiments conducted in this timeframe.

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