EVALUATION OF MC2010 MODELS FOR PRESTRESSED SFRC BEAMS SUBJECTED TO SHEAR

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Abstract

In the fib Model Code 2010 (MC2010), new provisions are given for calculating the ultimate limit state of fibre reinforced concrete elements. To assess the given design equations for the shear capacity of prestressed SFRC beams, the experimentally observed shear behaviour of long-span prestressed beams is compared with respect to the two analytical models provided in MC2010. Eventually, the feasibility of the models to predict the experimentally observed shear capacity is evaluated.

Keywords: steel fibres, prestressed beams, shear design, Model Code 2010, analytical modelling

1. Introduction

In order to reduce the labour-intensive parts of the production procedure of prestressed prefabricated beams, several viable solutions to replace mild steel rebars by steel fibres has have been investigated in the past by researchers [1-3]. Although the technique has proven to work, a wide application of steel fibres as shear reinforcement has been hold back due to the lack of appropriate formulations, which can be used by engineers to design prestressed steel fibre reinforced concrete (SFRC) beams.

In the framework of an on-going research project, an experimental programme is carried out to investigate the shear capacity of real-scale SFRC beams. Based on an earlier study [4] at the Magnel Laboratory for Concrete Research, Ghent University, it was found that a relatively small amount of steel fibres is able to carry the shear load comparable to a beam with the minimum conventional stirrup configuration according to Eurocode 2 [5]. Further investigations are done by the authors in order to enlarge the experimental dataset. For this additional series of four beams, the beam cross-section, prestress level and fibre type and dosages were differentiated with respect to the first series of beams.

Furthermore, the experimentally obtained shear capacity of all tested beams are compared with respect to the formulations given in the new Model Code 2010 [6] to see for which

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cases the given equations yield sufficient accuracy for estimating the shear capacity of prestressed SFRC beams.

2. Materials & methods

2.1 Test specimens

The experimental programme is carried out on two different series of I-shaped prestressed beams. A first series of three beams are 900 mm high and have a web thickness equal to 80 mm (B-900). The additional series of four beams have a height of 1000 mm and a web thickness equal to 90 mm (B-1000). 11 and 17 strands with an initial prestress force equal to 1350 kN are placed in beams of series B-900 and B-1000 respectively. In figure 1, the beam cross-sections are shown together with the configuration of prestress strands and upper mild steel. The beams with steel fibres (and without traditional web shear reinforcement) are compared with specimens which have traditional stirrup reinforcement (TR) as well as with plain concrete (PC) beams without fibres or stirrups as shear reinforcement. Table 1 gives an overview of the experimental programme with the specimen designation and a short description.

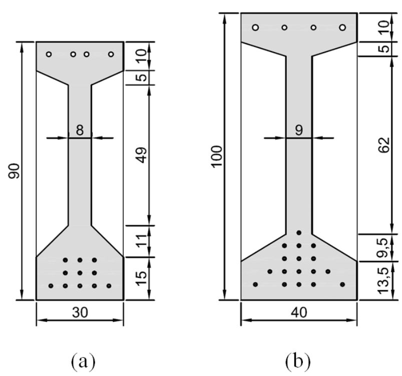

Fig. 1: Cross-sections of beams B-900 (a) and B-1000 (b) (dimensions in cm)

Tab.1: Test matrix

Beam	Height [mm]	Span length [m]	Fibre content [kg/m³]	Stirrups
B-900-PC	900	10.3	0	-
B-900-TR	900	10.3	0	2 Ø8 mm, @ 300 mm
B-900-40	900	10.3	40	-
B-1000-PC	1000	17.6	0	-
B-1000-20	1000	17.6	20	-
B-1000-40	1000	17.6	40	-
B-1000-TR	1000	17.6	0	2 Ø8 mm, @ 270 mm

2.2 SFRC mixes

For the two tested series, different SFRC mixes were used. For the first three beams, a traditional mix (Mix 1) is used without any optimisation regarding the effect of high dosages of steel fibres. For the second series of four beams, an optimisation of the mix

(Mix 2) is done in order to assure a good workability and compaction ability when higher dosages of steel fibres are intended to be used. In this way, a slump equal to at least 200 mm (an S4 according to EN 206-1:2000) up to a maximum fibre dosage of 60 kg/m³ has to be achieved while the air-content shall not be higher than 2 %. Two types of high strength hooked-end fibres were used. The DRAMIX cold-drawn wire fibre types RC-80/60-BP and RC-80/30-CP are used for mixes 1 and 2 respectively. Both types of fibres have a high tensile strength (≥ 2000 MPa) in order to avoid fibre rupture when used in combination with higher strength concretes. Table 2 summarizes the concrete compositions for both mix 1 and 2. Table 3 gives the most important properties of the concrete.

Tab.2: SFRC mix compositions

Constituent [kg/m³]	**Mix 1**	**Mix 2**
CEM I 52.5 R/HES	385	390
Fly ash	-	60
Water	200	190
W/C-ratio	0.474	0.400
Sand 0/1	-	202
Sand 0/2	805	-
Sand 0/4	-	674
Crushed limestone 2/6	200	257
Crushed limestone 6/14	-	566
Crushed limestone 6/20	820	-
Superplastifier	1.92	2.61

The parameters which characterize the post-cracking tensile behaviour of SFRC are derived from three-point bending tests according to the European standard EN-14651 [7]. This test on a notched specimen yields a load versus crack mouth opening displacement (CMOD) curve from which residual stresses f_R are calculated at certain values of CMOD. At a CMOD value of 0.5, 1.5, 2.5 and 3.5 mm the residual flexural stresses are respectively denoted as f_{R1}, f_{R2}, f_{R3}, f_{R4}. According to the rigid-plastic model (MC 2010, chapter 5), a design value f_{Ftu} is derived based on the value of f_{R3} assuming an ultimate crack opening w_u equal to 2.5 mm (see section 4). For each beam, the material parameters are summarized in table 3.

Tab.3: Concrete properties for each beam

Beam	**Fibre type**	**V_f [kg/m³]**	**$f_{cm,cyl}$ [N/mm²]**	**f_{R3} [N/mm²]**
B-900-PC	-	0	52.6	0.00
B-900-40	RC-80/60-BP	40	52.0	2.65
B-900-TR	-	0	52.6	0.00
B-1000-PC	-	0	68.5	0.00
B-1000-20	RC-80/30-CP	20	41.0	2.92
B-1000-40	RC-80/30-CP	40	70.4	5.80
B-1000-TR	-	0	73.2	0.00

2.3 Test setup

To assess the designed shear load, three point bending tests were conducted with a shear-span to depth ratio (a/d-ratio) equal to 3.4 and 2.5 for beams B-900 and B-1000 respectively. The load is applied by an hydraulic jack with a capacity of 1000 kN for the first four beams B-900 and a capacity of 2000 kN for beams B-1000. By means of Linear Variable Displacement Transducers (LVDT's), the deflection of the beam is measured at the position of the point-load, at midspan and at the supports.

The test setup configurations for both beams series B-900 and B-1000 are shown in figure 2. For beams B-900 the beams are supported 300 mm from the physical end of the beam. Although this is the most relevant way to support, this configuration forces the cracks to change their propagation direction at the end-section of the beam due to the presence of the rectangular end block and hence, the a/d-ratio is not clearly related to the crack formation. In order to avoid this possible restriction for the beams series B-1000, the supports were moved 300 mm outwards the I-section.

While the first series of beams B-900 were tested at one side only, the larger span of the beams series B-1000 allowed to conduct a test at both beam ends. The maximum shear load obtained for phase 1 and phase 2 are in this paper further denoted with A en B respectively.

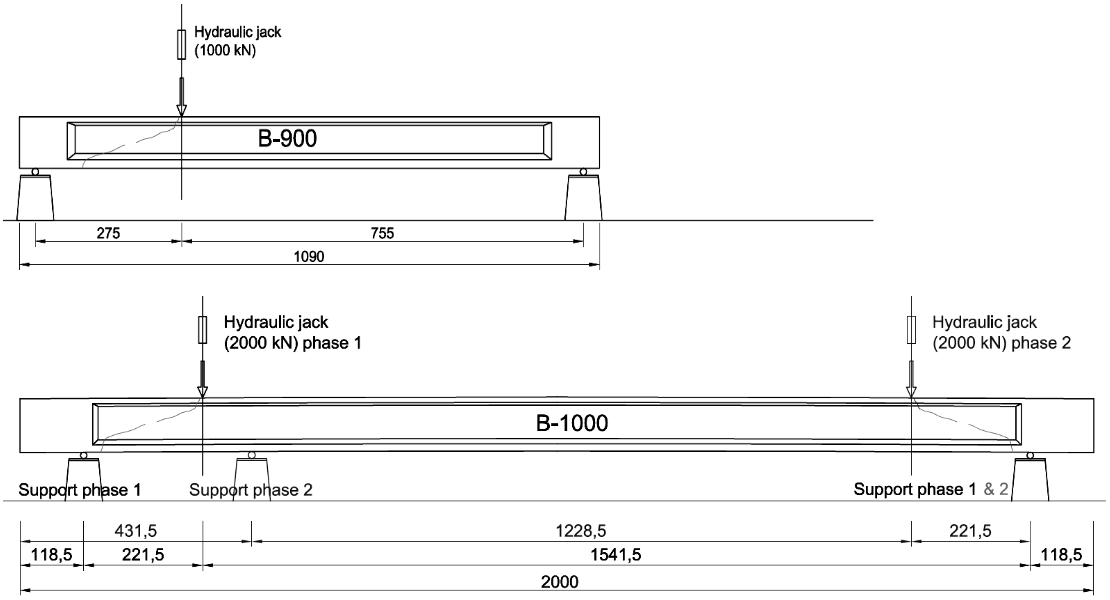

Fig. 2: Shear test setups for beams B-900 and B-1000

3. Results

In figure 3, the measured load versus deflection is shown for beam series B-900. The load-deflection curves for both phases of beams series B-1000 are shown in figure 4.

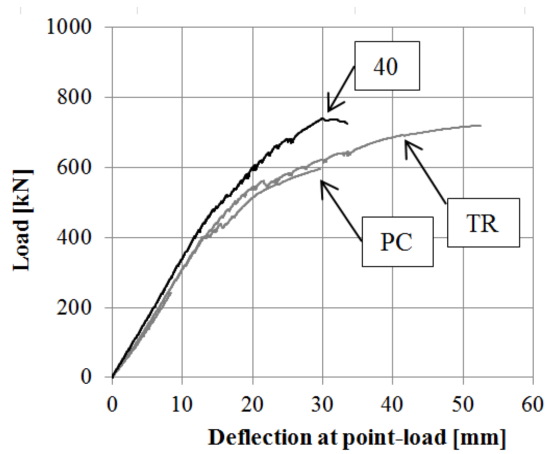

Fig. 3: Load-deflection curves for beams B-900

When there is no shear reinforcement (B-900-PC), the beam failed at a load of 597 kN. By adding 40 kg/m³ of steel fibres (RC-80/60-BP) an increase in ultimate shear capacity (740 kN) equal to 24% is observed. This observed increase has the same order of magnitude (+21%) as in the case of traditional stirrups, where a maximum shear load of 720 kN is observed. Hence, for this configuration, the shear capacity is high enough to replace all mild steel rebars as shear reinforcement.

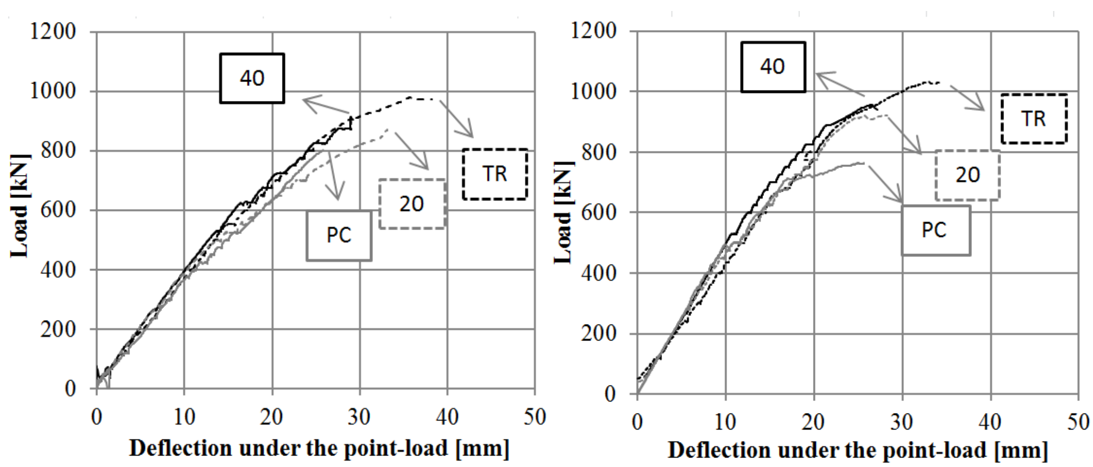

Fig. 4: Load-deflection curves for beams B-1000 - left: phase 1 and right: phase 2

For the second series of beams B-1000 and for both phase 1 and 2, an increase of shear capacity was observed as a function of fibre dosage. The highest relative increase was observed when adding 20 kg/m³ with respect to the plain concrete (PC) beam (+14.2 %). A much smaller increase of shear capacity for the beam with 40 kg/m³ with respect to beam B-1000-20 was found (+4.3 %). This indicates that the increase of shear capacity is not a linear function of fibre dosage. In contrast to the first series of beams B-900, a dosage of 40 kg/m³ fibres (RC-80/30-CP) is not sufficient to replace traditional stirrups (two bars diam. 8 mm) when placed at 270 mm distance. An overview of the observed shear capacities for all of the tested beams (both phase 1 and 2 for series B-1000) is given in figure 5.

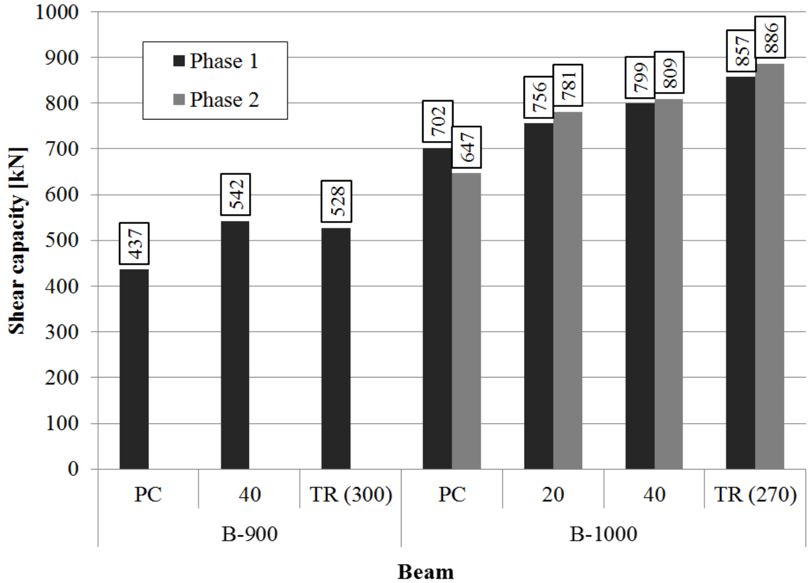

Fig. 5: Overview of shear capacity for all of the tested beams

4. Analytical models

For SFRC, Model Code 2010 provides an adaptation (eq. 1) to the shear design equation for the concrete contribution as it is well known from the Eurocode 2 provisions for shear. After cracking occurs, the shear capacity is mainly attributed to aggregate interlock and friction at the rough crack interface when the crack opening is controlled by the presence of a longitudinal reinforcement section. In a similar way, the shear load capacity of SFRC is enhanced by the effect of fibre pull-out capacity on the aggregate interlock phenomena. Therefore, an additional factor equal to 7.5 times the ratio of f_{Ftuk}/f_{ctk} is added. When using the design equations for evaluating the maximum experimental shear load, the partial material safety factors are set equal to one and instead of using characteristic or design values, average values of all material properties are adopted. Hence, the shear capacity of cracked SFRC is given by the following equation.

$$V_{F,1} = \left(0.18 \cdot k \cdot \left[100 \cdot \rho_l \left(1 + 7.5 \cdot \frac{f_{Ftum}}{f_{ctm}} \right) \cdot f_{cm} \right]^{1/3} + 0.15 \cdot \sigma_{cp} \right) \cdot b_w \cdot d \qquad (1)$$

with

k	size effect factor $= 1 + \sqrt{\frac{200}{d}} \leq 2$	[-]
d	effective depth of the cross-section	[mm]
ρ_l	longitudinal geometrical reinforcement ratio $= \frac{A_{sl}}{b_w d}$	[-]
f_{cm}	average cylinder compressive strength	[N/mm²]

f_{Ftum}	average value of the ultimate residual tensile strength	[N/mm²]
f_{ctm}	average tensile strength of the concrete	[N/mm²]
σ_{cp}	average stress acting on the cross section due to prestress	[N/mm²]
b_w	smallest width of the cross section in the tensile area	[mm]

The value of f_{Ftum} is needed to deal with the beneficial effect of steel fibres in a concrete matrix. Therefore, the post-cracking constitutive law of SFRC is simplified as a rigid plastic model as proposed in the Model Code 2010. All values of f_{Ftum} are calculated based on the residual flexural strength at a critical CMOD equal to 2,5 mm.

$$f_{Ftum} = \frac{f_{R3m}}{3} \tag{2}$$

As an alternative for the first shear design formulation, a second design equation (eq. 3) is proposed in the commentary section of the Model Code. This equation is proposed by Foster et al. [8] and is related to the assumptions made in the Modified Compression Field Theory (MCFT) as described by Vecchio & Collins [9] and the Variable Engagement Method (VEM) [8]. While the formulation of $V_{F,1}$ has a direct term related to the presence of a prestress, it is not straight forward implemented in the expression of $V_{F,2}$. Therefore, it is a necessary condition to approach the shear problem at a higher level of approximation (Level III) [6]. The shear capacity is then given by iteratively solving the set of equations 3-6.

$$V_{F,2} = \left(k_v \sqrt{f_{cm}} + f_{Ftum} \cot q\right) \cdot b_w \cdot z \tag{3}$$

with

$$k_v = \frac{0.4}{1 + 1500\varepsilon_x} \tag{4}$$

$\theta = 29° + 7000\varepsilon_x$, inclination angle of compression strut (5)

The strain at mid-depth of the cross-section ε_x will be calculated by eq. 6:

$$e_x = \frac{\frac{M_i}{z} + 0.5 \cot q \cdot V_i - A_p f_{p0}}{2\left(E_s A_s + E_p A_p + E_c A_c\right)} \geq -0.002 \tag{6}$$

with

M_i	flexural moment in the middle of the shear span (i^{th} iteration)	[Nmm]
V_i	calculated shear capacity (i^{th} iteration)	[Nmm]
E	modulus of elasticity *	[N/mm²]

A	cross-section *	[mm²]
f_{p0}	initial prestress	[N/mm²]
z	internal lever arm (equal to 0.9 d)	[mm]

* steel (s), prestress strand (p) and concrete (c)

For prestressed beams the inclination angle of shear cracks are much lower than in the case for traditional beams, as also expressed by eq. (5) and (6).

For beams with stirrups, the additional shear load contribution of the present stirrups V_s is calculated by means of equation 7.

$$V_s = \frac{0.9 \cdot d \cdot \cot(q)}{s} \cdot A_{sw} \cdot f_{ym} \tag{7}$$

with

s	distance between stirrups	[mm]
A_{sw}	cross-section of stirrups, crossing an inclined crack	[mm]
θ	inclination angle of the shear crack	[°]
f_{ym}	average yield stress of shear reinforcement (580 MPa)	[N/mm²]

To allow for a correct comparison between the two model approaches, Vs is calculated according to the variable inclination angle method (θ between 23° and 45°), whereas θ is taken according to eq. (5).

The total calculated shear capacity V_{cal} is then given by eq. 8:

$$V_{cal} = V_F + V_s \tag{8}$$

5. Model verification

To evaluate the accuracy and applicability of the considered analytical models herein, a comparison is made between the calculated shear capacity based on average values and the experimentally observed maximum shear load. In order to assess the given models an overview of the ratio of V_{test} and V_{cal} is given in table 4. Figure 6 shows a parity diagram for both models.

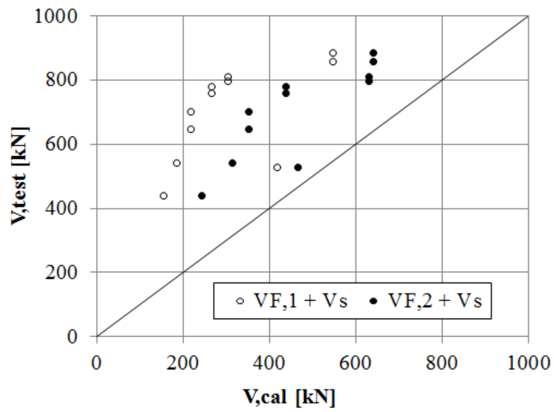

Fig. 6: Parity diagram for both models

Tab.4: Model verification

Beam	V_{test} [kN]	$V_{F,1} + V_s$ V_{cal} [kN]	$V_{F,1} + V_s$ V_{test}/V_{cal} [-]	$V_{F,2} + V_s$ V_{cal} [kN]	$V_{F,2} + V_s$ V_{test}/V_{cal} [-]
900 - PC	437	156	2.80	244	1.79
900 - 40	542	186	2.92	315	1.72
900 - TR	528	419	1.26	468	1.13
1000 - PC	702	219	3.20	353	1.99
1000 - PC	647	219	2.95	353	1.83
1000 - 20	756	268	2.83	439	1.73
1000 - 20	781	268	2.91	439	1.78
1000 - 40	799	306	2.61	633	1.26
1000 - 40	809	306	2.65	633	1.28
1000 - TR	857	549	1.56	643	1.33
1000 - TR	886	549	1.61	643	1.38
		Average	**2.48**	**Average**	**1.56**
		C.O.V.	**27%**	**C.O.V.**	**19%**

Regarding the values given in table 4 and the parity diagram in figure 6, it can be seen that the second approach is much less conservative than the first shear model and tends to be more accurate both in terms of predicted load and coefficient of variation.

The main difference between the two models is that the first model is based on empirical findings and that the effect of steel fibres is only affecting aggregate interlock. In contrast to the first model, in the second model, fibre contribution is quite similarly treated as it is the case for mild steel rebar since a more direct cross-bridging ability of fibres is taken into account to estimate the shear capacity.

A second difference lies in the fact that to implement an effect of prestress, for the second model, a level III of approximation has to be taken into account.

6. Conclusions

The addition of a limited amount of steel fibres can significantly increase the shear capacity of prestressed beams and in some cases it is possible to remove the minimum amount of required mild steel rebars. By adding for about 40 kg/m³ of high strength hooked-end fibres it is possible for the considered specimens in this work to replace a shear reinforcement ratio of about 0.42% (steel grade equal to BE500).

Although fibres are effective as shear reinforcement, the current design models provided by the Model Code 2010 show great differences in predicting the experimental shear capacity. It is proven that when a small increase in calculation effort is undertaken, a much less conservative approach is achieved and the influence of fibres is implemented in a more realistic way. This led to a more accurate prediction of the shear capacity of prestressed SFRC beams.

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