PREDICTION OF THE SHEAR STRENGTH OF STEEL FIBRE REINFORCED CONCRETE BEAMS

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Abstract

Shear failure is brittle and is initiated by diagonal tensile stresses created in the tension zone. Designers usually avoid this type of failure by using shear reinforcement. Due to their ability in enhancing the tensile strength of concrete, restraining and retarding cracks and increasing the shear friction resistance, steel fibres can replace the conventional and sometimes laborious shear reinforcement specially in thin members.

This paper review some of the available equations for the prediction of cracking and shear strength of fibre reinforced concrete beams. These equations were applied to 44 beams to predict the first crack shear stress and 100 beams to predict the shear strength. A method is proposed to estimate the first crack and shear strength of fibre reinforced concrete beams. The proposed methods proved their superiority over the previously proposed methods. It was statistically shown that the tensile strength of concrete affect the cracking shear stress more than the shear strength which is more effected by the shear span / effective depth ratio. The contribution of the steel fibres to the shear strength is more than that at the onset of first shear cracking.

Keywords: Cracking, fibres, prediction, shear, steel, strength.

1. Introduction

The action of external loads on reinforced concrete members usually create axial forces, shear forces, bending moments and torsion or a combination of two or more actions. Shear failure is of a brittle type and initiated by diagonal tensile stresses created in the tension zone. Designers usually avoid this type of failure by using shear reinforcement. Due to their ability in enhancing the tensile strength of concrete, restraining and retarding cracks and increasing shear friction resistance, steel fibres can replace the conventional and sometimes laborious shear reinforcement specially in thin members [1,2].

Sharma [3] tested seven reinforced concrete beams in two groups, the first group was with stirrups and the second group was with steel fibres. He found that steel fibres were effective in increasing shear strength of reinforced concrete beams, and proposed the following equation for predicting the unit shear strength V_u of fibre reinforced concrete (FRC) beams:

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$$v_u = (2 f_{spfc} / 3)(d / a_v)^{0.25} \quad (1)$$

f_{spfc} = splitting strength of fibrous concrete, d = beam effective depth and a_v = shear span.

Narayanan and Darwish [4] tested 49 FRC beams in two groups, one with stirrups and the other with steel fibres. The variables included, fibres volume fraction, fibres aspect ratio, concrete compressive strength, longitudinal reinforcement ratio and shear span / effective depth. They found that the improvement in shear strength due to steel fibres addition is similar to the improvement gained by using stirrups. They proposed the following equation for predicting cracking shear stress, v_{cr} :

$$v_{cr} = 0.24 f_{spfc} + 20\rho .d / a_v + 0.5F \quad (2)$$

f_{spfc} = splitting strength of fibrous concrete and the following equation was proposed to estimate it:

$$f_{spfc} = f_{cuf} /(20 - \sqrt{F}) + \sqrt{F} + 0.7 \quad (3)$$

f_{cuf} = cube strength of fibrous concrete, ρ = longitudinal reinforcement ratio and F is fibre reinforcing index and is equal to:

$$F = b_f .V_f .L_f / D_f \quad (4)$$

b_f is a fibre shape factor = 0.5 for plain smooth fibres, 0.75 for hooked fibres and 1.0 for crimped fibres, V_f , L_f and D_f are the fibres volume fraction, length and diameter of steel fibres respectively.

Another equation was also proposed for predicting the unit shear strength of FRC beams:

$$v_u = e[0.24 f_{spfc} + 80\rho .d / a_v] + v_b \quad (5)$$

e is a nondimensional factor representing the effect of the arch action and is equal to:

$$e = 1.0 \quad \text{for} \quad a_v/d < 2.8 \quad (6a)$$

$$e = 2.8d / a_v \quad \text{for} \quad a_v/d \geq 2.8 \quad (6b)$$

v_b = pullout stress of steel fibres, calculated by:

$$v_b = 0.41\tau_b \times F$$

(7)

τ_b = interfacial bond strength of steel fibres with concrete and equal to 4.15 MPa [5].

AlTaan and AlFeel [6] used the regression analysis to find an equation for predicting the cracking shear stress (v_{cr}) from the experimental results of 38 previously tested FRC beams [4,7], the following equation was found:

$$v_{cr} = [\sqrt{f'_c} + 260\rho(d / a_v)e + 4.4F_{be}.V_f .L_f / D_f]/8.5 \quad (8)$$

e is a nondimensional factor representing the effect of the arch action, which was considered to be:

$e = 1.0$ for $a_v/d < 2.5$ (9a)

$e = 2.5d / a_v$ for $a_v/d \geq 2.5$ (9b)

F_{be} = bond efficiency factor = 1.0 for plain fibres, 1.2 for hooked fibres and 1.3 for duoform and crimped fibres.

AlTaan and AlFeel [6] proposed a method for predicting the shear strength of FRC beams, based on the method proposed by Zsutty [8] and Placas and Rangan [9] for normal reinforced concrete beams:

$$v_{uc} = (10\rho f_c'.d / a_v)^{1/3} \quad \text{for } a_v / d > 2.5 \tag{10a}$$

$$v_{uc} = (160\rho f_c')^{1/3} (d / a_v)^{4/3} \quad \text{for } a_v / d < 2.5 \tag{10b}$$

f_c' = cylinder strength of plain concrete. The external bending moment caused by the shear force assumed to be counteracted by the internal bending moment:

$$v_{uc}.b.d.a_v = 0.85 f_c'.a.b(d - a / 2)$$
(11)

The depth of the compression stress block, a, can be found from the above equation as shown below:

$$a/d = 1 - \sqrt{1 - 2.353(v_{uc} / f_c')(a_v / d)} \tag{12}$$

To estimate the contribution of the steel fibres to the unit shear strength, the following equation was used:

$$v_{uf} = \sigma_{tu} (h - c)/d \tag{13}$$

h= overall beam depth, c = neutral axis depth = a / β_1 [10], β_1= ratio of the depth of the compression stress block to the neutral axis depth c and σ_{tu} = post cracking tensile strength of the fibrous concrete [11]:

$$\sigma_{tu} = 0.5\tau_b \times V_f .L_f / D_f \tag{14}$$

τ_b = interfacial bond strength of steel fibres with concrete and considered to be 4.15 MPa [5]. The shear strength found from Equation (10) was added to that found from Equation (13) to get the total shear strength v_{ut}:

$$v_{ut} = v_{uc} + v_{uf} \tag{15}$$

This value was then substituted in Equation (12) to get a better estimate of (a), then the steps were repeated until the value of (a) converge to an acceptable degree of accuracy.

Another Equation was also derived for predicting the shear strength (v_u) from the experimental results of 89 reviously tested FRC beams [1,4,7,12-14], using the regression analysis, the following equation was found:

$$v_u = [1.6\sqrt{f_c'} + 960\rho(d / a_v)e + 8.5F_{be}.V_f .L_f / D_f]/9$$
(16)

The effect of steel fibres on the behaviour and shear strength of high strength reinforced concrete [15-18] and self compacted reinforced concrete beams are also reported [19-22].

The aim of this study is to review and assess some of the previously proposed shear strength equations for FRC beams and to propose equation for predicting the cracking and shear strength of FRC beams without shear reinforcement.

1.1 Assessment of the previously proposed equations for cracking shear stress

Experimental data of the cracking shear stress were collected from previously tested 44 FRC beams [4,7,12] whose geometrical and materials properties are shown in Table (1). The methods proposed by Narayanan and Darwish [4] and AlTaan and AlFeel [6] were used for predicting the cracking shear stress and the results are as shown in Table (2).

Tab. 1: Range of the properties of the tested FRC beams for cracking shear stress

Properties	H (mm)	B (mm)	L_f (mm)	D_f (mm)	V_f (%)	f'_c MPa	f'_{cf} MPa	ρ (%)	ρ' (%)
Minimum	150	75	20	0.3	0.25	24.2	29.1	1.56	0.0
Maximum.	225	150	40	0.5	3.0	52.64	63.6	5.67	3.75

2. Proposed method for the cracking shear stress

The experimental data for the cracking shear stress of the 44 FRC beams [4,7,12] were used in a regression analysis between the shear stress and the factors influencing it, such as the tensile strength of plain concrete, post cracking tensile strength of fibrous concrete, the shear span / effective depth ratio and reinforcement ratio [23]. The following equation was obtained:

$$v_{cr} = [1.015\sqrt{f'_c} + 84\rho(d/a_v).e + 1.87\sigma_{tu}]/7 \quad (17)$$

e is a nondimensional factor representing the effect of the arch action taken as the proposed value in reference [6]. The correlation coefficient for the above equation is 0.994 and the standard error is 0.02. The first term in the above equation is similar to that recommended by the ACI Code [10] which is an indirect measurement of the contribution of tensile strength of plain concrete to cracking shear stress. The second term is the contribution of the arch action, which is not so effective at this stage since the beam behaviour is almost like a homogenous uncracked beam. The third term is the contribution of fibre concrete to cracking shear stress. It is worth noting that at this stage, fibres are not fully mobilized as will be shown later for the ultimate stage which showed a higher value. Figure (1) shows the histogram of the calculated / measured cracking shear stress with the normal distribution curve. The results shown in Table (2), indicate that the proposed method is better than the other two methods [4,6] in terms of the average value and skewness, α. Skewness is an assessment of the symmetry of normal distribution and a zero or small value indicate a symmetric distribution about the average, while a negative value indicate a left tail longer than the right one and a positive value indicate a right tail longer than the left one.

Tab. 2: Results of the ratios of the predicted to the measured cracking shear stress

Method	Average	S.D.	C.O.V.%	α	Results within ± 10%
Narayanan and Darwish [4]	0.95	0.1112	11.7	-0.43	80%
AlTaan and AlFeel [6]	0.99	0.1515	15.3	-0.105	68%
Present method	1.0	0.118	11.8	-0.066	80%

S.D., standard deviation, C.O.V. , coefficient of variation and α is the skewness.

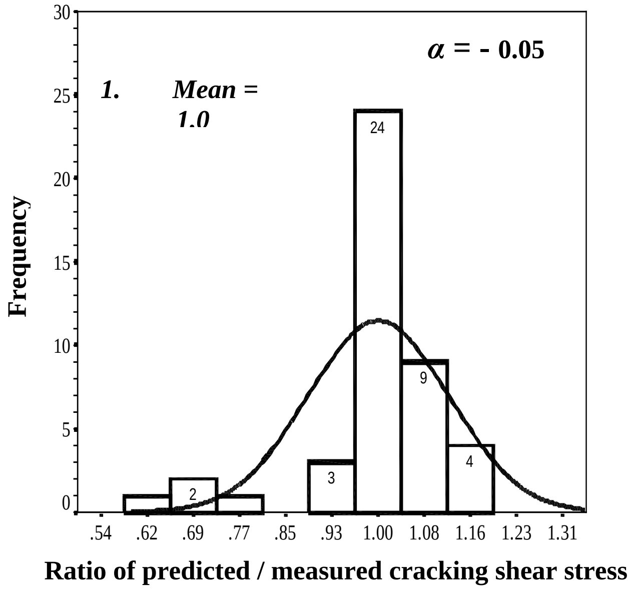

Fig. 1: Histogram of the ratios predicted / measured cracking shear stress (proposed method)

Table (3) show the Standardized coefficients for the three terms in the above equation. It is shown that ($\sqrt{f_c'}$) has a highest value, which is an index for cracks initiation and it has the highest effect on the cracking shear stress. The post cracking tensile strength of fibrous concrete has the second effect on the cracking shear stress, which is approximately 81% of the first term and the second term has the least effect 19% of the first term at this stage.

Tab. 3: Statistical properties of the terms of Equation (17)

Variable	$\sqrt{f_c'}$	$\rho(d/a_v).e$	σ_{tu}
Standardized Coefficient(β)	0.513	0.1	0.418
Significance	0.000	0.003	0.000

2.1 Assessment of the previously proposed equations for the shear strength

Experimental data for the shear strength were collected from previously tested 100 FRC beams [1,4,7,12-14,24-26] whose geometrical and materials properties are shown in Table (4). The methods proposed by Sharma [3], Narayanan and Darwish [4] and AlTaan and AlFeel [6] are used for predicting the shear strength and the results shown in Table (5).

Tab. 4: Range of the properties of the tested FRC beams for the shear strength

Properties	h mm	B Mm	L_f (mm)	D_f (mm)	V_f (%)	f'_c (MPa)	f'_{cf} (MPa)	ρ (%)	ρ' (%)
Minimum	100	50	19	0.25	0.22	24.2	28.28	0.2	0.0
Maximum	375	152	40	1.335	2.0	52.64	57.5	2.39	3.75

2.2 Proposed methods for the shear strength (Method 1)

This method is a modification of the first method proposed by AlTaan and AlFeel [6]. The first change is adding the contribution of the compression steel to the flexural strength in equations (11-12), although this effect is not paramount. The second change is using the post cracking tensile strength of fibrous concrete σ_{tu} instead of using equation (14) which includes the interfacial bond strength τ_b of steel fibres with concrete. The value of the bond strength is highly variable in the range between 2.3 -8.3 MPa [23] which depends on the method of testing (whether a flexural or a direct tension), number of fibres (single or a group and the direction of the fibre with the applied stress), fibres texture, besides other factors such as, matrix constituents and aggregate size. In the absence of experimental results, the following equation proposed by Ahmed and Pama [27] can be used:

$$\sigma_{tu} = 0.3753 f_{rf} - 0.806 \tag{18}$$

f_{rf} = flexural strength of fibrous concrete. Equation (12) was used first to estimate the shear strength of the reinforced concrete beam. If a beam is single reinforced, equations (11-12) are used to calculate the neutral axis depth. If a beam is double reinforced, the following equation was used to calculate the neutral axis depth:

$$V_u . a_v = v_{uc} .b.d.a_v = 0.85 f'_{cf} .a.b\left(d - a/2\right) + A'_s . f''_s \left(d - d'\right) \tag{19}$$

A_s'= area of the compression steel, f_s''= effective compression steel stress and d'= depth of the compression steel. Equation (13) is then used to estimate the contribution of steel fibres to the shear strength. The shear stress found from Equation (10) was added to that found from Equation (13) to get the total shear strength v_{ut}, this value was then substituted in Equation (12) to get a better estimate of (a) and the steps were repeated until the value of (a) converge to an acceptable degree of accuracy. The procedure described above was applied to the 100 FRC beams [1,4,7,12-14,24-26]. The average ratio of the predicted / measured shear strength = 0.993, S.D. = 0.138, C.O.V. = 13.9% and skewness = 0.198.

3. Method 2

Experimental data for the shear strength were collected from the previously tested 100 FRC beams [1,4,11,14-16,24-26] and used in a regression analysis between the shear strength and the factors influencing the shear strength such as tensile strength of concrete, post cracking tensile strength of fibrous concrete, the shear span / effective depth ratio and the reinforcement ratio [23]. The following equation was obtained:

$$v_u = [1.155\sqrt{f_c^{'}} + 624\rho(d/a_v).e + 4.571\sigma_{tu}]/7 \qquad (20)$$

The correlation coefficient for equation (20) is 0.994 and the standard error is 0.043. The first term in equation (20) is about 14% more than that in equation (17) and to that recommended by the ACI Code [9] which is an indirect measurement of the contribution of the tensile strength of plain concrete. The second term representing the arch action and equal to 7.4 times that at the first cracking stage. This large contribution of the arch action at this stage is due the full mobilization of this action at this stage. The third term is the contribution of the steel fibres to the shear strength which equal to 2.44 times that at the first cracking stage due to the full mobilization of the steel fibres. The results shown in Table (5), indicate that this proposed method is better than the other three methods [3,4,6] in terms of the average value, coefficient of variation and the results within ± 10% of the average value. Figures (2,3) show the histogram and the normal distribution curves for the two proposed methods.

Table (6) show the standardized coefficients for the three terms in the equation (20). It is shown that the arch action effect have the greatest effect at this stage. The steel fibres has the second effect, similar to the first cracking stage and is equal to 83% of the effect of the arch action. The term ($\sqrt{f_c^{'}}$) has the least effect at this stage and equal to 67% of the arch action since this effect is vanished when the cracks are well developed prior to the ultimate stage.

Tab. 5: The ratios of the predicted to the measured cracking shear stress

Method	Average	S.D.	C.O.V.%	A	Results within ± 10%
Sharma [4]	0.88	0.237	26.9	-0.206	32%
Narayanan and Darwish [5]	0.90	0.15	16.7	-0.27	45%
AlTaan and AlFeel [7][1]	0.91	0.177	19.5	0.333	29%
AlTaan and AlFeel [7][2]	0.96	0.124	12.9	0.155	57%
Present porposed method 1	0.993	0.138	13.9	0.198	54%
Present proposed method 2	1.01	0.125	12.3	0.171	61%

Tab. 6: Properties of the independent variables for the shear strength Equation

Variable	$\rho(d/a_v).e$	σ_{tu}	$\sqrt{f_c'}$
Standardized Coefficients (β)	0.422	0.351	0.286
Significance	0.000	0.000	0.000

Figure 4, shows the predicted shear strength using the first proposed method with the measured shear strength of references [4,13] for a shear span/effective depth ranging from 1.5 to 3.5 and different types of steel fibres, volume fraction and aspect ratio. Figure (4) also shows the close agreement between the predicted and the measured shear strength.

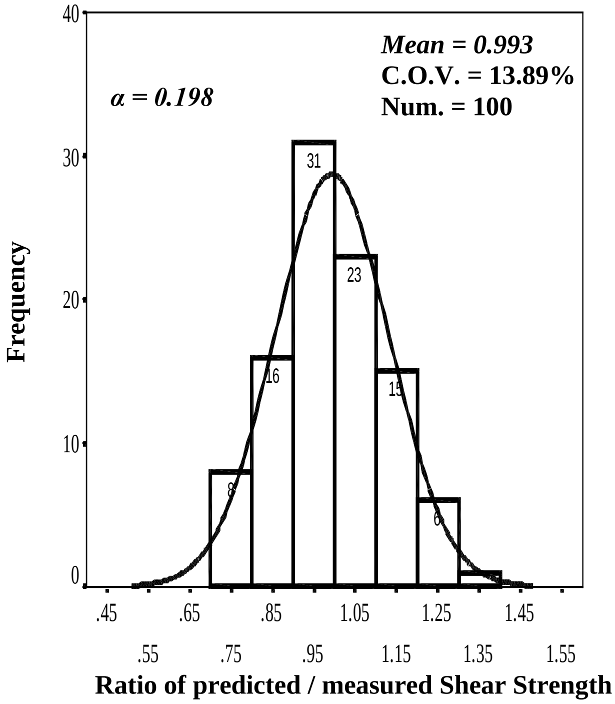

Fig. 2: Frequency diagram of the ratios predicted / measured shear strength (first proposed method)

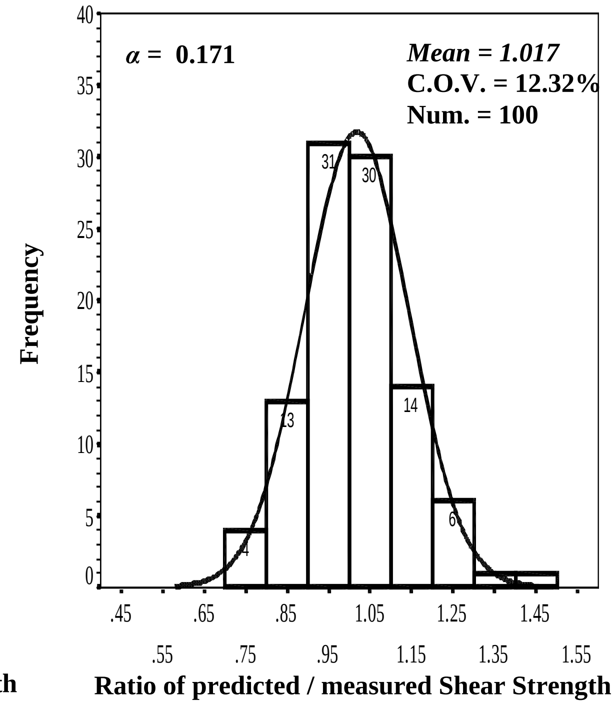

Fig. 3: Frequency diagram of the ratios predicted / measured shear strength (second proposed method)

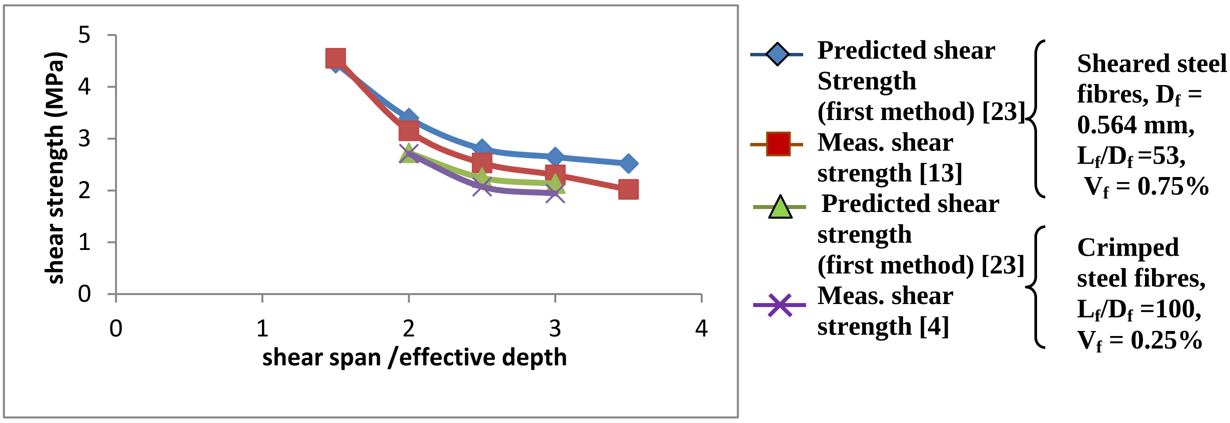

Fig. 4: Comparison between predicted and measured shear strength

4. Conclusions

The proposed methods for the cracking shear stress and shear strength of the investigated FRC beams showed better prediction than the other previously proposed methods. The statistical analysis showed that the tensile strength of concrete affect the cracking shear stress more than the shear strength which is more effected by the shear span / effective depth ratio. The contribution of the steel fibres to the shear strength is more than that at the onset of first shear cracking. More experimental data will refine the proposed methods that can be used as a design guide.

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