PREDICTION OF THE TORSIONAL STRENGTH OF STEEL FIBRE REINFORCED CONCRETE MEMBERS

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Abstract

Torsion failure is of a brittle type and is initiated by shear stresses created over the whole cross-section and exceeds the tensile strength of concrete. Designers usually avoid this type of failure by using torsional reinforcement. Due to their ability in enhancing the tensile strength of concrete, restraining and retarding cracks, increasing the shear friction resistance and imparting ductility, steel fibres can replace the conventional and sometimes laborious torsional reinforcement specially in thin members.

This paper review the available equations for the prediction of the torsional strength of fibre concrete members and those for the torsional strength of fibre reinforced concrete members. A method was then proposed to predict the torsional strength of fibre reinforced concrete members which assumed to be composed of three components; strength provided by plain concrete before cracking, strength provided by the steel fibres after cracking and the torsional strength provided by the torsional reinforcement. The proposed method was applied to 25 previously tested fibre reinforced concrete members and showed good agreement with the experimental results.

More experimental data are required to refine the proposed method which can be used as a design guide.

Keywords: Cracking, fibres, prediction, steel, strength, torsion.

1. Introduction

Internal forces like the axial forces, shear forces, bending moments and torsional moments are created in reinforced concrete members due to the action of the external loads. Depending on the nature and distribution of the external loads, the created internal forces usually created individually or a combination of two or more forces. Torsion is created when the resultant of the external forces does not pass through the shear centre of the cross section. Torsion failure is of a brittle type and is initiated when the tensile stresses created by the shear stresses created over the whole cross-section exceeds the tensile strength of concrete. Designers usually avoid this type of failure by using torsional reinforcement.

Steel fibres have the ability in enhancing the tensile strength of concrete, restraining and retarding cracks, imparting ductility to concrete and increasing the shear friction resistance

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[1,2], and can therefore replace the conventional and sometimes laborious torsion reinforcement specially in thin members.

Test results showed that adding steel fibres, impart a post cracking torsional strength and ductility, this improvement depends on the fibre type, volume fraction and aspect ratio [3-10].

Torsional strength of fibre concrete beams

Narayanan and Goloosalar [3] tested square concrete specimens under pure torsion. Three mix proportions were used with 0.7% volume fraction of (38.1×0.254mm) steel fibres. They concluded that adding steel fibres, increased the torsional strength of concrete and the specimens exhibited ductility prior to failure. The following equation was used for predicting the cracking torsional strength T_{cr}:

$$T_{cr} = \omega x^2 y\left(0.24\sqrt{f_{cuf}}\right) \tag{1}$$

ω is a cross section shape factor and = 0.208 for square sections, x and y are the shorter and longer side of the cross section respectively and f_{cuf} is the cube strength of fibrous concrete (FC). The above equation predicts about one-third to half of the observed cracking torque.

The authors used the skew bending theory to estimate the torsional strength T_u of FC specimens:

$$T_u = \left(0.85 f_{rf}\right) x^2 . y/3 \tag{2}$$

f_{rf} is the flexural strength of fibrous concrete. Equation (2) predicts about 40-60 percent of the torsional strength.

Narayanan and Green [4] conducted experimental tests on FC beams subjected to bending, torsion and combined bending and torsion up to failure. Four concrete mixes were designed, and fibers volume fractions ranging between 0.5 to 2 percent with aspect ratio of 50 were used. They found that steel fibres increase the torsional strength of FC beams and proposed the following equation for predicting it:

$$T_u = \left(\frac{1}{2} - \frac{x}{4.5y}\right) x^2 . y . f_{sp} \tag{3}$$

f_{sp} = splitting strength of plain concrete, the following equation which was proposed by Marshall [11] was used for calculating it:

$$f_{sp} = f_{cu}/15.5 + 0.8 \tag{4}$$

Mansur [5] tested three groups of 100 mm square FC beams and 100, 150 and 200 mm long under pure torsion. The beams were reinforced with hooked end fibres (30× 0.4mm) and 0.75% volume fraction He used the skew bending theory for analyzing the failed beams:

$T_u = \phi . f_{rf} . x^2 . y / 3$ (5)

Mansur [5] used his test results and those of Mansur and Paramasivam [6] to derive the strength reduction factor ϕ and found to be = 0.7.

Narayanan and Palanjian [7] tested 28 FC beams subjected to pure torsion. They varied the fibre type, fibre aspect ratio, fibres volume fraction and cross section. They proposed the following equation for predicting the cracking T_{cr} and torsional strength T_u of FC beams:

$$T_{cr} = T_p [1 + K_c (b_f V_f L_f / D_f)] \quad (6)$$

$$T_u = T_p [1 + K_u (b_f V_f L_f / D_f)] \quad (7)$$

T_p is the torsional strength of plain concrete beams, they used the following equation for predicting it after assuming that the tensile strength of concrete = $0.37\sqrt{f_{cu}}$:

$$T_p = 0.13 x^2 . y \sqrt{f_{cu}} \quad (8)$$

f_{cu} = cube strength of plain concrete, K_c and K_u are empirical constants = 0.3 and 0.42 respectively, b_f is a fibre shape factor = 0.5 for smooth fibres, 0.75 for hooked fibres and 1.0 for duoform and crimped fibres, V_f , L_f and D_f are the volume fraction, length and diameter of the fibres respectively. They reported an increase in the torsional strength up to 100% due to the presence of steel fibres.

Tegos [8] conducted an experimental study to explore the effect of steel fibres on the behavior and strength of cylindrical concrete specimens using different concrete strength, fibres volume fraction and aspect ratio. The experimental results showed that the increase in the torsional strength depends on the fibres volume fraction, fibres aspect ratio and the concrete compressive strength. The following equation was proposed for predicting the torsional strength of FC:

$$T_u = (L_f / D_f)^{3/2} . V_f . T_p / 15 \quad (9)$$

T_p is the torsional strength of the plain concrete member.

Rao and Seshu [9] tested twenty FC beams subjected to pure torsion, the test results showed that the steel fibres increase the torsional strength, rigidity and toughness. The following equation was proposed for predicting the torsional strength of FC beams and showed good agreement with the test results:

$$T_u = \omega x^2 y f_{tf} \quad (10)$$

ω is a shape factor = [0.5 – x/(6y)], f_{tf} = biaxial tensile strength of FC and the following equation was proposed for its estimation:

$$1/ f_{tf} = 1/ f'_{cf} + 1/ f_{spfc} \quad (11)$$

f'_{cf} = cylinder strength of FC.

AlTaan and AlFeel [10] proposed a method for predicting the cracking and torsional strength of FC beams. The method is based on the thin-walled tube analogy recommended by the ACI Committee 318-14 [12]. The torsional strength assumed to be composed of that for plain concrete prior to cracking T_p and the contribution of the steel fibres after cracking T_f :

$$T_u = T_p + T_f \tag{12}$$

Prior to cracking the member is considered as a tube with wall thickness t = 0.75 A_{cp} / P_{cp} as adopted by ACI code [12], where A_{cp} = area of the gross concrete section, and P_{cp} is the perimeter of the section. The shearing stress τ due to torsion T can be calculated using the principles of mechanics of materials as:

$$\tau = T/(2A_o.t) \tag{13}$$

A_o = (y-t)(x-t) area enclosed by a line around the tube at the mid thickness of the wall. In the case of pure torque, the principal tensile stress equal to the shearing stress and was assumed to be $0.9f_{sp}$ which is in general agreement with the value used by Hsu [13] and T_p equal to:

$$T_p = 0.9 f_{sp} \left(2A_o.t\right) \tag{14}$$

f_{sp} = splitting strength of plain concrete and equation (4) was used for its estimation. Equation (14) was used for predicting the torsional cracking strength of 68 FC beams from references [7,8]. The average predicted / measured strength was 0.98 with a coefficient of variation of 11.8%.

After cracking, a FC beam subjected to pure torsion can be idealized as a space truss. The torsional strength of this truss as recommended by the ACI [12] can be written as follow:

$$T = 2A_o.A_t.f_y.\cot\theta / s \tag{15}$$

where A_t = area of one leg of the closed stirrup, f_y = yield strength of the reinforcement, s = stirrups spacing, and θ = angle of inclination of the cracks spiraling the member (perpendicular to the principal tensile stress direction) assumed equal to 45 [12,14]. The steel fibres through the wall thickness is transformed to equivalent stirrups $A_t = \eta_o.V_f.s.t$ with an effective stress equal to $\sigma_f = \tau_u.L_f / D_f$, η_o = orientation factor for randomly distributed fibres which was taken as 0.41 [15] and τ_u = interfacial bond strength of the steel fibres with concrete and the equation proposed by Souroshian and Lee [15] was used in this investigation:

$$\tau_u = 2.62 - 0.0036N_f \tag{16}$$

N_f =number of fibres per unit area, which is:

$$N_f = \eta_o.V_f /(\pi.d_f^2 / 4) \tag{17}$$

Substituting the area of the stirrups and the effective stress into equation (15), the following equation was obtained for T_f:

$$T_f /(2A_o.t) = \eta_o.\tau_u.V_f L_f / D_f \tag{18}$$

The left hand side of equation (18) is the unit torsional stress (τ_f) provided by the steel fibres and the right hand side is the post cracking tensile strength of fibrous concrete (σ_{ut}). The above procedure was applied to 98 FC beams [4,5,7,8,16] under pure torsion. The average ratio of the calculated to the measured values was 1.028 and a coefficient of variation of 15%.

Rjoub and Musmar [17] used also the thin-walled tube analogy to derive a prediction equation for SFC beams under pure torsion. The effective tensile strength of concrete in pure torsion, equation (11), in addition to other parameters are introduced and derived from the experimental results.

2. Torsional strength of fibre reinforced concrete beams

Craig et al. [16] tested nine reinforced concrete beams under pure torsion, all with same longitudinal reinforcement, two specimens were without fibres and the other seven were with different volume fraction of steel fibres and stirrups spacing. They proposed the following two equations for predicting the torsional strength of FC beams:

$$T_u = 1.56\left(x^2 + 10\right)y\sqrt[3]{f_r{}^2} \quad (19)$$

$$T_u = 1.67\left(x^2 + 10\right)y\sqrt[3]{f_{sp}{}^2} \quad (20)$$

For FRC beams, the following equation for predicting the torsional strength of FRC beams was proposed:

$$T_u = \alpha_t . x_1 . y_1 . A_t . f_y / s + \alpha_c \left(x^2 . y\sqrt{f_c'}\right) \quad (21)$$

α_t is a factor that depends on the type and amount of the fibres, x_1 and y_1 are the shorter and longer side of the stirrup centre line and α_c is a factor that depends on the fibre pull out force P_f which is:

$$\alpha_c = 2 + 0.12 P_f / \sqrt{f_c'} \quad (22)$$

A good agreement was found between the experimental and calculated values.

Mansur and Lim [18] tested fifteen rectangular reinforced concrete beams, included some unreinforced FC beams, FC beams reinforced longitudinally only, and FC beams reinforced longitudinally and transversely. The experimental results showed that the steel fibres can be best utilized in improving the torsional strength when the beams are reinforced longitudinally and transversely.

Narayanan and Kareem-Palanjian [19] tested twenty four concrete beams with partial or full replacement of longitudinal and or transverse reinforcement by an equivalent volume fraction of steel fibres to study the effect of steel fibres on the behavior and strength of reinforced concrete beams subjected to torsion. The test results indicated that the first cracking torque increased substantially compared to similar beams provided with conventional reinforcement. The torsional strength was increased up to 60% when the stirrups were partially or totally replaced by steel fibres. They proposed the following equation for calculating the torsional strength of FRC beams reinforced longitudinally and transversely:

$$T_u = T_p + T_f + T_r \tag{23}$$

T_p = torsional strength of plain concrete which was estimated using equation (8), T_f = torsional strength provided by the steel fibres and the thin-walled tube analogy was used [20] to calculate it:

$$T_f = 0.22[(x_o.y_o/(x_o + y_o)] \times x.y.\sqrt{f_{cu}} \times b_f.V_f.L_f/D_f \tag{24}$$

x_o = 5x/6, shorter side of the centre line of the tube thickness assumed equal to $x/6$ and y_o= y -$x/6$. T_r = torsional strength provided by the longitudinal and transverse reinforcement, which was estimated by the proposed equation of Victor and Muthukrishnan [21]:

$$T_r = \lambda..x_1.y_1.A_t.f_{yt}/s \tag{25}$$

f_{yt} = yield strength of the transverse reinforcement and λ is a constant that depends on the longitudinal and transverse reinforcement ratios and stirrups dimensions [21]:

$$\lambda = (1 - k_1)\left[0.2m + \sqrt{m}\left(\frac{0.45y_1}{x_1} - \frac{s}{x_1 + y_1}\right)\right] \tag{26}$$

k_1 is a constant that depends on the arrangement of the longitudinal steel [21]:

$$k_1 = (2c_1/h)(f_{y,bottom}/f_{y,top}) \tag{27}$$

c_1 = distance between the cross section centroid and the centroid of the longitudinal reinforcement which is equal to zero for symmetrically reinforced sections and m is the ratio of the longitudinal to the transverse reinforcement strength [21]:

$$m = \frac{V_l \cdot f_y}{V_t \cdot f_{yt}} = \frac{A_l/(x.y)}{2A_t(x_1 + y_1)/(s.x.y)} \times \frac{f_y}{f_{yt}} \tag{28}$$

V_l = percentage volume of longitudinal reinforcement, f_y = yield strength of the longitudinal reinforcement, V_t = percentage volume of transverse reinforcement and A_l = area of the longitudinal reinforcement

In the present investigation, the above described procedure was applied to 25 FRC beams tested previously under pure torsion [16,18,19,22-24] whose materials and geometrical properties are shown in Table (1). The average ratio of the calculated / measured torsional strength = 1.247, a coefficient of variation = 13.3%, skewness = -0.605 and 24% of the results are within 10± of the equality value [25]. The results indicated that the method overestimated the experimental torsional strength.

Tab. 1: Range of the properties of the investigated FRC beams under pure torsion

Range	h mm	b mm	L_f mm	D_f mm	V_f %	f_c' MPa	f_y MPa	f_{yt} MPa	s mm	V_t^* %	V_l^{**} %
Minimum	155	85	25	0.3	0.5	16.0	310	310	45	0.18	0.25
Maximum	310	152	50	0.5	2.0	41.0	460	400	178	1.34	1.16

* Volumetric percentage ratio of transverse torsional reinforcement,
** Volumetric percentage ratio of longitudinal torsional reinforcement.

The aim of the present investigation is to propose a method for predicting the torsional strength of FRC beams subjected to pure torsion.

3. Proposed method for the prediction of torsional strength of FRC beams

The torsional strength of FRC beams under pure torsion assumed to be composed of three parts as mentioned in equation (23). The first component is the torsional strength provided by plain concrete and equation (14) proposed by AlTaan and AlFeel [10] was used in this investigation. The mobilization of steel fibres starts after the onset of cracking, and equation (18) proposed by AlTaan and AlFeel [10] was used in this investigation in the following form:

$$T_f = (2A_o.t)\sigma_{ut} \quad (29)$$

The thickness of the tube wall (t) was assumed equal to (b/6) instead of the value recommended by the ACI Committee 318-14 [12] and used by AlTaan and AlFeel [10], this latter value gave a better prediction of the torsional strength of FRC beams [25]. The equation proposed by Ahmed and Pama [26] was used in this investigation for estimating the post cracking tensile strength of fibrous concrete, since it is easier than the expression in equation (18) which involve five parameters:

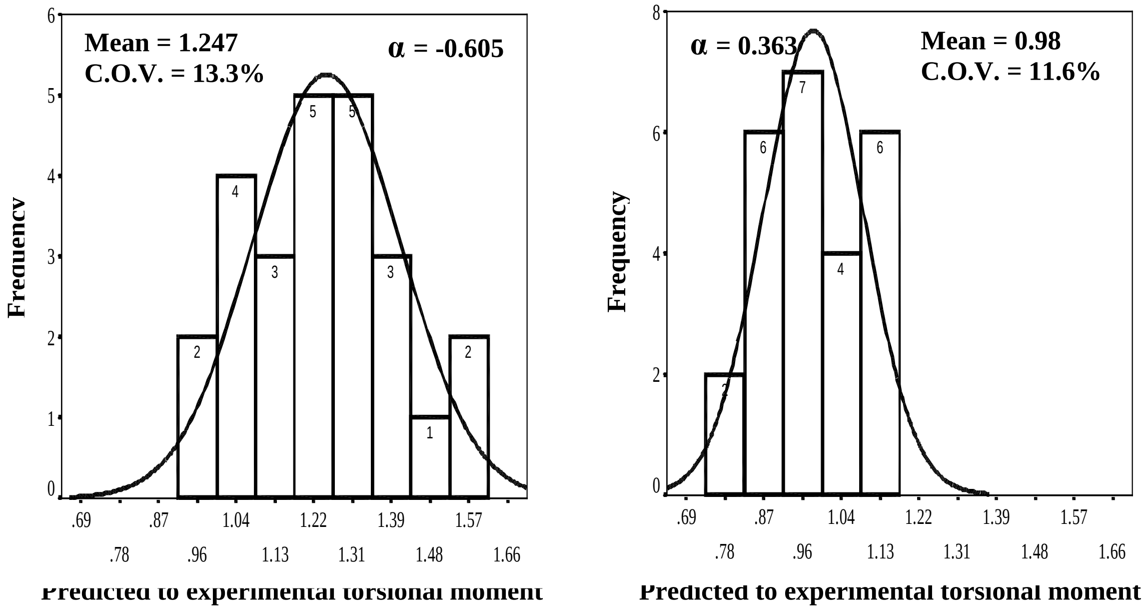
$$\sigma_{ut} = 0.3753 f_{rf} - 0.806 \quad (30)$$

Equations (25-28) proposed by Victor and Muthukrishnan [21] were used for calculating the torsional strength provided by the longitudinal and transverse reinforcement T_r .

The proposed procedure described above was used for predicting the torsional strength of the 25 FRC beams whose details are shown in Table (1). The average ratio of the predicted / experimental torsional strength = 0.98, a coefficient of variation = 11.6%, skewness = 0.363, and 52% of the results were within ± 10% of the equality value [25].

Figure (1) shows the histograms for the ratios of the predicted / experimental torsional moment for the method proposed by Narayanan and Palanjian [20] and the present proposed method. The Figure shows that the proposed method is better than that proposed in Reference [20] in terms of the average, coefficient of variation and skewness which show a more uniform and narrower distribution about the average.

Figures (2,3) show the variation of the experimental and the predicted torsional moment using the present proposed method versus the volume fraction of two types of fibres [16,18]. The steel fibres used were of the crimped and Dramix type. Acceptable degree of correlation between the predicted and the experimental torsional moment is clear from the two figures.



(a) Narayanan and Palanjian [20] (b) Present proposed method

Fig. 1: Histograms of the ratio predicted to experimental torsional moment

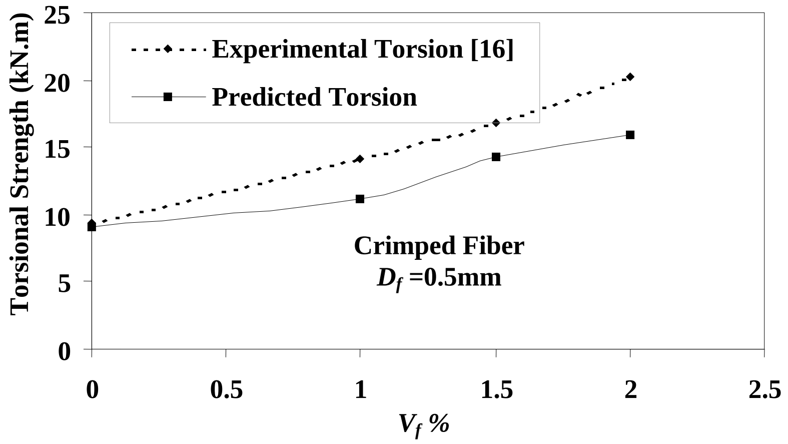


Fig. 2: Effect of fibres volume fraction on the torsional strength

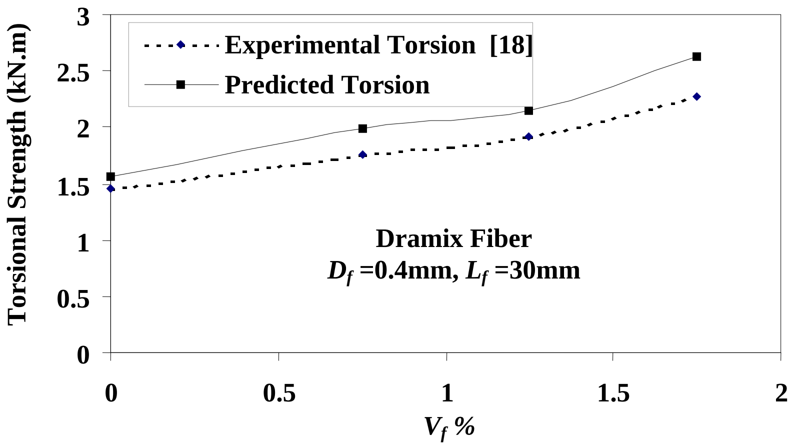


Fig. 3: Effect of fibres volume fraction on the torsional strength

4. Conclusions

A method is proposed for predicting the torsional strength of FRC beams and showed a reliable degree of accuracy. The method identify the effect of the influencing variables, like the concrete strength, steel fibres properties and longitudinal and transverse torsional reinforcement. More experimental results will refine the proposed method which will serve as a design guide.

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