MODELLING OF STEEL FIBRE REINFORCED CONCRETE (SFRC) UNDER DYNAMIC COMPRESSION

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Abstract

Steel fibre reinforced concrete (SFRC) structures are often subjected to impact and blast loads leading to the high strain rate effects thus necessitating investigations for the dynamic characterization of the material. The present paper focuses on the modelling aspects of SFRC based on Split Hopkinson Pressure Bar (SHPB) compression tests. Circular disk shaped specimens of SFRC with different percentages of steel fibres were prepared and tested for the strain rate range of 25 to 125 s^{-1}. Stress-strain curves were plotted for plain concrete and SFRC specimens at different strain rates. The models available for plain and SFRC composites are discussed. A unified equation using the concept of reinforcing index is proposed for the estimation of dynamic increase factor (DIF) for strength of SFRC. A model for the DIF of critical strain is also developed. The DIF predicted by reinforcing index based model matches adequately well with the experimental values. Further, rate dependent analytical model is also developed for the prediction of complete stress-strain curve of SFRC under dynamic compression loading. The predicted curves show good conformance with the experiments.

Keywords: Steel fibres; Concrete; Strain rate; Stress-strain curve; DIF

1. Introduction

Steel fibres, because of their economic viability and relatively higher values of ductility and elastic modulii, are widely used as fibre reinforcing material in concrete. The steel fibre reinforced concrete (SFRC) is the extensively used concrete mix in industrial and civil buildings of strategic importance.

Studies in the past were conducted on the dynamic compression response of plain concrete and SFRC. For the plain concrete/mortar specimens tested under varying strain-rates using Split Hopkinson Pressure Bar (SHPB), the main feature of strain rate and compressive

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strength relationship was a two stage strain-rate dependent behaviour – the DIF for strength increases up to a critical strain-rate value with certain rate and then beyond the critical strain-rate value, the DIF for strength increases rapidly with increasing strain-rate. Some authors treated the two stage behaviour by using a single nonlinear relationship for the strain rate and DIF (strength); others relate the behaviour using a set of linear equations [1, 2]. For the SFRC specimens, in addition to the above, the stress-strain relationship also becomes a function of geometric and material properties of steel fibres. In the next section, available models for plain and SFRC specimens are reviewed.

2. Analytical models

Based on experimental tests conducted on concrete/mortar specimens using SHPB, some typical empirical formulae for predicting the DIF (strength) have been developed in the past. One of the most commonly used empirical equations for the DIF (strength) were given by the Comite Euro-International du Beton (CEB) [3]. The DIF (strength), for the two stage strain-rate dependent behaviour of concrete/mortar, was given by:

$$\left.\begin{aligned} \text{DIF} &= \frac{f_{cd}}{f_{cs}} = \left|\frac{\dot{\varepsilon}}{\dot{\varepsilon}_s}\right|^{1.026\,\alpha_s} && \text{for } \dot{\varepsilon} \le 30s^{-1} \\ \text{DIF} &= \frac{f_{cd}}{f_{cs}} = \gamma_s\left|\frac{\dot{\varepsilon}}{\dot{\varepsilon}_s}\right|^{\frac{1}{3}} && \text{for } \dot{\varepsilon} \ge 30s^{-1} \end{aligned}\right\} \quad (1)$$

where $\dot{\varepsilon}$ is the strain-rate in s^{-1}; f_{cs} and f_{cd} are the unconfined compressive strength under quasi-static and dynamic loading, respectively, and $\gamma_s = 10^{(6.156\alpha_s - 2.0)}$; $\alpha_s = 1/(5 + 9f_{cs}/f_{co})$; f_{co} = 10 MPa; $\dot{\varepsilon}_s = 30 \times 10^{-6}$ s^{-1}. The transition for the two stage behaviour of DIF with respect to the strain-rate was found to be at a strain-rate value of 30 s^{-1}. Some other models based on power law distribution were also developed. The constitutive equation to describe the DIF (strength) of the compressive strength of concrete was proposed by Fujikake *et al.* [4] as given by:

$$\text{DIF} = \left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_s}\right)^{0.006\left[\log\left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_s}\right)\right]^{1.05}} \quad (2)$$

A strain rate dependent power law model of DIF (strength) based on the data collected from previous studies was given as [5]:

$$\text{DIF} = 0.05\left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_0}\right)^{0.13} + 0.9 \quad (3)$$

where, $\dot{\varepsilon}_0 = 1\ s^{-1}$

Ross et al. [6] and Tedesco and Ross [7] carried out a series of SHPB tests for different concrete strengths and a two-stage strain-rate dependent behaviour was observed. The transition for the two stage behaviour occurred at a strain-rate of 63.1 s^{-1} beyond which the DIF increases relatively more rapidly with increase in the strain-rate. The equations are:

$$\left.\begin{aligned} \text{DIF} &= 0.00965\log\dot{\varepsilon} + 1.058 \ge 1.0 && \text{for } \dot{\varepsilon} \le 63.1\ s^{-1} \\ \text{DIF} &= 0.758\log\dot{\varepsilon} - 0.289 \le 2.5 && \text{for } \dot{\varepsilon} \ge 63.1\ s^{-1} \end{aligned}\right\} \quad (4)$$

Grote [8] tested the cement mortar specimens on SHPB at strain-rates ranging from 250 to 1700 s^{-1} and proposed following equations for DIF:

$$\left.\begin{aligned} \text{DIF} &= 0.0235\log\dot{\varepsilon} + 1.07 && \text{for } \dot{\varepsilon} \le 266\ s^{-1} \\ \text{DIF} &= 0.882(\log\dot{\varepsilon})^3 - 4.4(\log\dot{\varepsilon})^2 + 7.22\log\dot{\varepsilon} - 2.64 && \text{for } \dot{\varepsilon} \ge 266\ s^{-1} \end{aligned}\right\} \quad (5)$$

Li and Meng [9] conducted experimental and numerical studies on solid and annular mortar samples and gave the following equations:

$$\left.\begin{array}{ll} DIF = 0.03438\,(3+\log \dot{\varepsilon})+1 & \text{for } \dot{\varepsilon} \leq 100\ s^{-1} \\ DIF = 1.729(\log \dot{\varepsilon})^2 - 7.1372 \log \dot{\varepsilon} + 8.5303 & \text{for } \dot{\varepsilon} \geq 100\ s^{-1} \end{array}\right\} \quad (6)$$

Further refinement of the above model (Eq. 6) was suggested by Zhang *et al*. [10] wherein the values of the constants (i.e. 0.03438 and 3) for strain-rates ≤ 100 s^{-1} were replaced by 0.0258 and 4 respectively. This change was carried out to counter a local drop of DIF (strength) within the transitional range of strain-rate from 10^{-3} to 10^{-4} s^{-1}.

Katayama *et al*. [11] presented another DIF (strength) model with variation given by logarithmic trend and the equation was given as:

$$DIF = 1.021 - 0.05076 \log \dot{\varepsilon} + 0.2583(\log \dot{\varepsilon})^2 \quad (7)$$

A model which is a combination of power law and logarithmic equations – for strain-rates up to 300 s^{-1}, was given by Ngo [12]:

$$\left.\begin{array}{ll} DIF = \left|\frac{\dot{\varepsilon}}{\dot{\varepsilon}_s}\right|^{1.026\alpha} & \text{for } \dot{\varepsilon} \leq \dot{\varepsilon}_1 \\ DIF = A_1 \ln(\dot{\varepsilon}) - A_2 & \text{for } \dot{\varepsilon} \geq \dot{\varepsilon}_1 \end{array}\right\} \quad (8)$$

where, A_1 = 0.9866 – 0.0044 f_{cs}; A_2 = 2.1396 – 0.0128 f_{cs}; α = 1/(20 + f_{cs}/2); $\dot{\varepsilon}_1 = 0.0022 f_{cs}^2 - 0.1989 f_{cs} + 46.137$.

Investigations on the modelling of rate dependent stress-strain behaviour of SFRC are quite limited as compared to the modelling of rate dependent behaviour of plain concrete. Wang et al. [13] studied the rate dependent stress–strain behaviour of SFRC and proposed a model for stress-strain curve based on Weibull function. The stress-strain relation can be expressed as:

$$\sigma = E\varepsilon\left[\exp\left(\frac{\varepsilon}{F_0}\right)^m\right] \quad (9)$$

where, E is the elastic modulus of SFRC, ε is the strain, the shape parameter m and the scale parameter F_0 are related to volume fraction (V_f) and strain rate $\dot{\varepsilon}$ is:

$$m = (1 - 0.02V_f)(1 + 0.04 \ln \dot{\varepsilon})$$
$$F_0 = (1 + 0.03V_f)(1 + 1.25 \ln \dot{\varepsilon})$$

DIF (strength) and DIF (toughness) models were given by Hao and Hao [14] for SFRC containing spiral fibres. However, different models were given for different percentage of steel fibres for both DIF (strength) and DIF (toughness).

From the literature review, it is clear that the models for SFRC are quite limited. Also, most of the available models for plain concrete involve more than one equation for prediction of DIF at different strain rates. Therefore, an effort is made in this paper to present a unified model based on SHPB experiments. SFRC circular disk shaped specimens with different percentage of hooked steel fibres were prepared and tested under dynamic compression using SHPB for the strain rate range of 25 to 125 s^{-1}.

3. Experimental setup

The schematic setup of SHPB is given in Fig. 1. The details of the experimental setup are given in authors earlier papers [1, 2]. However, for brevity and completeness, only the equations are given here. The time histories of strain, strain rate and stress in the specimen at time *t* can be obtained from the following equations:

$$\varepsilon(t)=\frac{2C_b}{L}\int_0^t \varepsilon_r(\omega)d\omega, \tag{10}$$

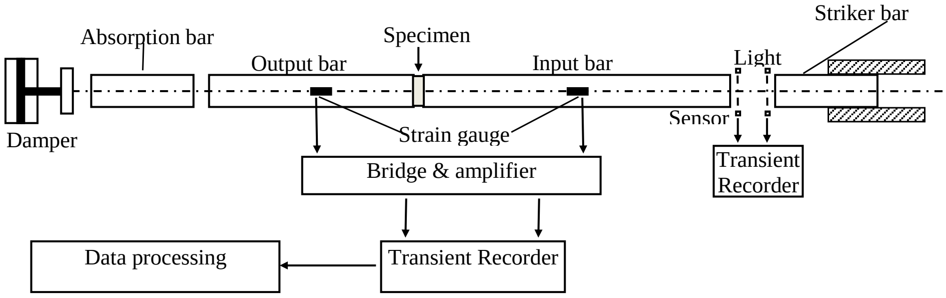
$$\dot{\varepsilon}(t)=\frac{2C_b}{L}\varepsilon_r(t), \text{ and} \tag{11}$$

$$\sigma(t)=\frac{A_b E_b}{A_s}\varepsilon_o(t). \tag{12}$$

where ε_i, ε_r and ε_o are incident compressive strain wave, reflected wave and transmitted wave respectively; E_b is the Modulus of elasticity of Hopkinson bars; A_s and A_b are the cross sectional areas of specimen and Hopkinson bars respectively; C_b is the velocity of longitudinal waves in Hopkinson bars; L is the length of specimen. The strain rate and stress history can be obtained as:

$$\dot{\varepsilon}(t)=\frac{C_b}{L}\{\varepsilon_o(t)+\varepsilon_i(t)+\varepsilon_r(t)\}, \text{ and} \tag{13}$$

$$\sigma(t)=\frac{A_b E_b}{2A_s}\{\varepsilon_i(t)+\varepsilon_r(t)+\varepsilon_o(t)\}. \tag{14}$$



(a)

Fig.1: Schematic setup of SHPB.

4. Experimental program

4.1 Materials

Type 1 ordinary Portland cement was used in the preparation of concrete. Table 1 shows the details of plain concrete mix. Hooked ended steel fibre, were used in the preparation of SFRC. The properties of the fibres are given in Table 2.

4.2 Specimens

Three cylindrical specimens with ϕ73 mm diameter and an aspect ratio of 0.5 (length of specimen = 36.5mm) of each SFRC composite mix were cast for testing them at different strain rates. For each mix, three standard cylindrical specimens were used for quasi-static testing. The experimental results are given in Table 3.

Tab.1: Details of plain concrete mix used in the casting of SFRC composite specimens.

Material	Weight (kg/m^3)
Cement	520
Fine sand	586
Coarse aggregate (Nominal size = 10 mm)	850
Coarse aggregate (Nominal size = 5 mm)	315
Water (water cement ratio = 0.28)	145
Gli-110 (Super-plasticizer)	3 Liters
Retarder LD10	1.5 Liters

Tab.2: Properties of the steel fibres.

Fibre length (mm)	Fibre diameter (mm)	Tensile strength (MPa)	Elastic modulus (GPa)	Specific gravity	Shape
60	0.75 ϕ	1225	200	7.85	Hooked ends

Tab.3: Test results summary.

Mix ID	Static strength, f_{cs} (MPa)	Strain rate (s^{-1})	Dyn. Strength, f_{cd} (MPa)
M0 (Control)	64.5	111	100.5
		78	86.0
		36	75.3
M1 (1.2% steel fibres)	73.5	110	121.5
		75	96.0
		32	90.0
M2 (1.4% steel fibres)	74.4	112	128.4
		71	106.0
		24	89.0

5. Results and discussion

5.1 Analytical model for DIF (strength) and DIF (strain)

For determining the dynamic compressive strength, f_{cd}, an equation which is a function of strain rate and the percentage of steel fibres for predicting the DIF (strength) has been developed based on the experimental tests conducted on SFRC specimens. The concept of the reinforcing index, RI_v, which has been used in describing the static mechanical properties of SFRC [15] has been extrapolated to consider the steel fibres with varied geometrical/material properties, RI_v, as:

$$RI_v = \sum_i^n RI_{vi} \tag{15}$$

where, suffix i is used for fibre type. The expression obtained here is general and can be used for different combination of fibres. RI_{vi} is the value of reinforcing index, RI_v, for the i^{th} material given by:

$$RI_{vi} = v_{fi} \frac{k_i l_i}{d_i} \frac{E_i}{E_s} \tag{16}$$

where, v_{fi} is the volume fraction of fibres; k_i is the bond factor of fibres; l_i is the length of fibres; d_i is the diameter (or equivalent diameter for non-circular sections) of fibres; E_i and E_s are the modulus of elasticity of the material of i^{th} fibres and steel fibres respectively. The value of bond factor, k_i, for hooked-end steel fibres used in this study is taken as 1. Taking RI_v into consideration, the expression for the DIF (strength) of SFRC obtained through regression analysis of the experimental data is:

$$(\mathrm{DIF})_{strength} = Exp\left(0.0033\dot{\varepsilon}\left(1 + 0.27RI_v^{\;1.38}\right)\right) \tag{17}$$

The DIF (strength) of different SFRC composites obtained using Eq. (17) is plotted and compared with the experimentally observed DIF (strength) in Fig. 2. The error scatter is well within ±10% range. Similar expression can be obtained for DIF (strain):

$$(\mathrm{DIF})_{strain} = Exp\left(0.0018\dot{\varepsilon}\left(1 + 4.5RI_v^{\;0.38}\right)\right) \tag{18}$$

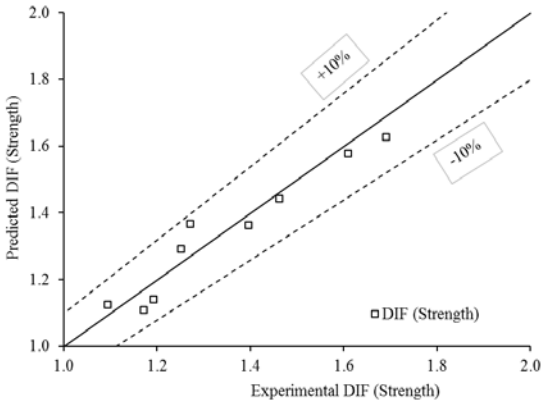


Fig. 2: Scatter in the prediction of DIF (strength) using Eq. (17).

5.2 Analytical model for stress-strain variation

An empirical model for the stress-strain variation, based on the experimental results obtained in the present study is also developed. The variation of stress over a strain range for different strain-rates can be given by rational equation as:

$$Y = \frac{AX + (B-1)X^2}{1 + (A-2)X + BX^2} \tag{19}$$

where A and B parameters are functions of strain rate, $\dot{\varepsilon}$; X and Y are the normalized strain and stress respectively, given by $Y = \frac{\sigma}{f_{cd}}$ and $X = \frac{\varepsilon}{\varepsilon_{cd}}$; where, f_{cd} and ε_{cd} are the peak strength and critical strain components of the stress-strain curve at a given strain rate $\dot{\varepsilon}$; σ is the stress corresponding to the strain ε. Firstly, the parameters A and B were obtained individually for different strain rates of each mix by minimizing the error between the experimental and predicted values of stress-strain curves. The values of parameters A and B thus obtained were used to develop models for these parameters in terms of the strain rate and the reinforcing index. The equations obtained for parameters A and B are:

$$A = 3.6Exp\left(0.003\dot{\varepsilon}\left(1 + 0.01{RI_v}^{0.82}\right)\right) \quad (20)$$

$$B = 0.22Exp\left(0.0127\dot{\varepsilon}\left(1 + 0.002{RI_v}^{0.82}\right)\right) \quad (21)$$

The stress-strain curves of different SFRC mixes used in the present study obtained at different strain rates are plotted in Figs. 3 and 4 along with the experimental stress-strain curves. The predicted curves match well with the experimental curves.

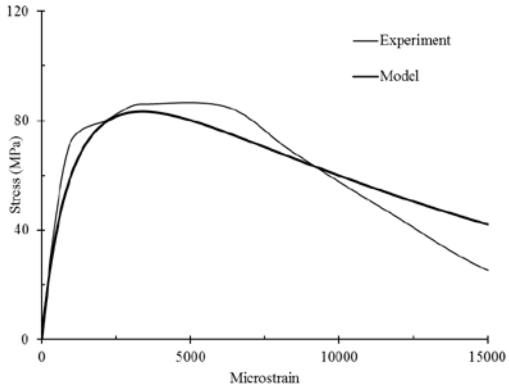
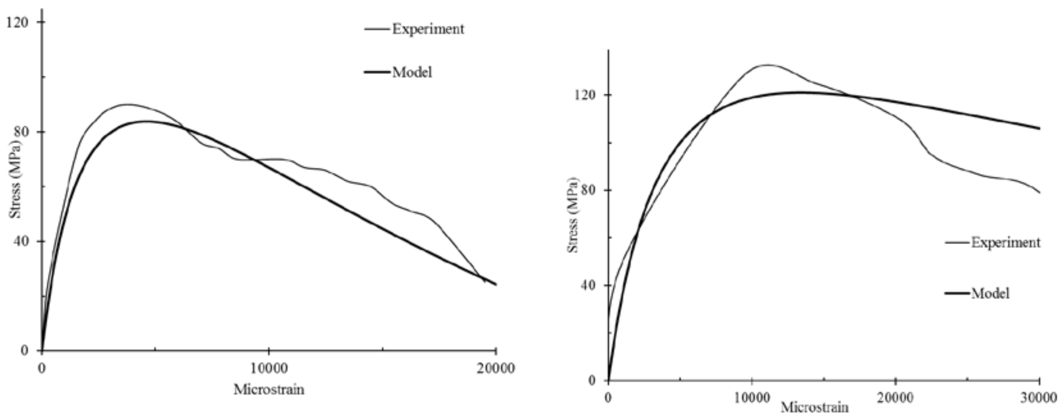


Fig. 3: Comparison of experimental and predicted stress-strain relationship for plain concrete mix at strain rate of 78 s^{-1}.



(a) (b)

Fig. 4: Comparison of experimental and predicted stress-strain relationship for fibre reinforced concrete: (a) Mix M1 at strain rate of 32 s^{-1}; (b) Mix M2 at strain rate of 112 s^{-1}.

6. Conclusions

An analytical model is developed for predicting complete stress-strain curves of SFRC composites. The parameters required for the model are function of reinforcing index of steel fibres and strain rate. The analytical model gives an adequate estimate of stress–strain curve of SFRC composites under moderate strain rate range.

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