EVALUATION OF FIBRE REINFORCED CONCRETE BEAM TEST RESULTS BASED ON THE EXAMINATION OF THE REAL FIBRE DISTRIBUTION

JUHÁSZ Károly Péter [1]

Abstract

The standard test methods of FRC (Fibre Reinforced Concrete) are bending beam tests. Due to the random distribution of the fibres there could be a big dispersion in the results. This large dispersion could be decreased by increasing the number of specimens and/ or the area of the representative cross section. In this article the possibility of decreasing the dispersion is researched with a method based on the real fibre distribution. The added fibres increase the residual flexural strength of the concrete, which is mainly influenced by the number of fibres on the cracked section, the distance of the intersection point of the fibre and the cracked section from the neutral axes, the anchorage length of the fibre and the angle between the fibre and the cracked section. From this data only the first two could be measured relatively easily and accurately. The number of fibres crossing the cracked section could be calculated by using mathematic statistics. By knowing the test results and the corresponding location of the fibres the result of the ideal distribution could be forecasted. By applying this method the dispersion caused by improper mixing may be avoided. In this article this method is being presented and verified by large number of test results.

Keywords: fibre distribution, bending beam test, evaluation of test results

1. Introduction

The calculation method of FRC could be separated to two parts: the smeared model, where the material model is defined by stress-strain or stress-crack width law [1, 2, 8, 9, 13], and the discrete model, where the fibres are modelled as discrete short length reinforcements (VEM: Variable Engagement Model [15]; DEM: Diverse Embedment Model [7]). In smeared model the FRC is considered as a composite material [6] and it is assumed that the distribution of the fibres is homogenous and oriented uniformly. The results of the beam tests have a big dispersion due to the relatively small reference area of the cross section and the inhomogeneity which is caused by the inaccurate mixing at the laboratory. Although the beam test method of the RILEM [10] was verified by round-robin analysis by several laboratories [11] the results still show that the improper mixing has a big influence. To increase the accuracy of the results, the size of the testing specimen could be increased, the same way as the round panel size had been increased in the new ATSM C1550 test [3];

[1] JUHÁSZ Károly Péter, 1111 Budapest, Műegyetem rkp. 3., Hungary, juhasz@szt.bme.hu

or the accuracy of the evaluation of the test results could be increased by choosing the appropriate probability distribution function (PDF) [4].

In this article a simple method is researched which could increase the accuracy by eliminating the effects of the improper mixing of the laboratory. This method could only be used in tests with localised cracks, where the material has a strain-softening law after cracking.

2. Effects of the distribution of fibres on the residual strength

The effects of the fibres on a bended cross-section mostly depend on the material and geometry of the fibre, the distance of the intersection point of the fibre and the cracked section from the neutral axes (a), the anchorage length of the fibre (b) and the angle between the fibre and the cracked section (c). Fibres not crossing the crack could have some effects on the crack process zone, but these fibres are neglected in this method (Fig. 1). From these three parameters the (a) could be measured relatively easily.

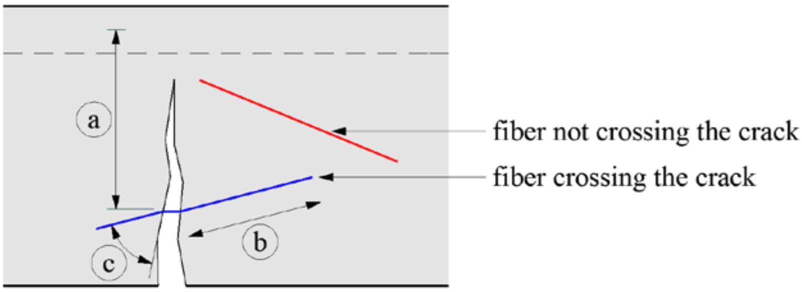

Fig. 1: Parameters of a discrete fibre

3. Experimental setup

Steel fibre reinforced beams were made with different dosages. The steel fibre reinforced concrete was mixed with concrete of three different strength classes (C25/30, C30/37, C35/45) with four kinds of dosages: 20, 25, 30 and 40 kg/m^3. Six beams were made of each kind. The property of the fibre could be seen in Table 1, and the matrix of the research could be seen in Table 2.

Tab.1: Types of the fibres

Name	Material	Form	Length/section	pieces/kg
F-1 Armofib® 150	steel	hooked, round cross section	50.7 mm/1 mm	3 126

Tab.2: Research matrix

Name of the series	Concrete type	Dosage [kg/m^3]
ST-concrete-dosage	C25/30; C30/37; C35/45	20; 25; 30; 40

The research was made according to the recommendations of RILEM TC 162 [10] with three points bending beam test with a notch in the middle, measuring the load and CMOD (Crack Mouth Opening Displacement) up to 3.5 mm. Test speed was 0.2 mm/min. The test was performed by a universal testing machine ZWICK Z150 in the Laboratory of

Department of Mechanics, Materials and Structures, Budapest University of Technology and Economics.

4. Evaluation of the test results

4.1 Reasons of high dispersion

Despite the proper and accurate production of the beams high dispersion appeared in the results (Fig. 2). This high dispersion occurred due to the combination of inherent inaccuracies of the production method. If the amount of the mixed concrete was prepared for more beams, and during the casting one of the beams got more fibre, than obviously the rest of the beams would get less. Thus the difference in the amount of fibre in the beams is increasing. During the casting of the beam fibres could also be distributed non-homogenously. Moreover, not only the number of the fibres, but also their location form the top surface matters in the beam test. In case of direct tensile test the location of fibres on the cracked surface has smaller effect on the result, nevertheless this test method is very difficult and generally not used in the industry.

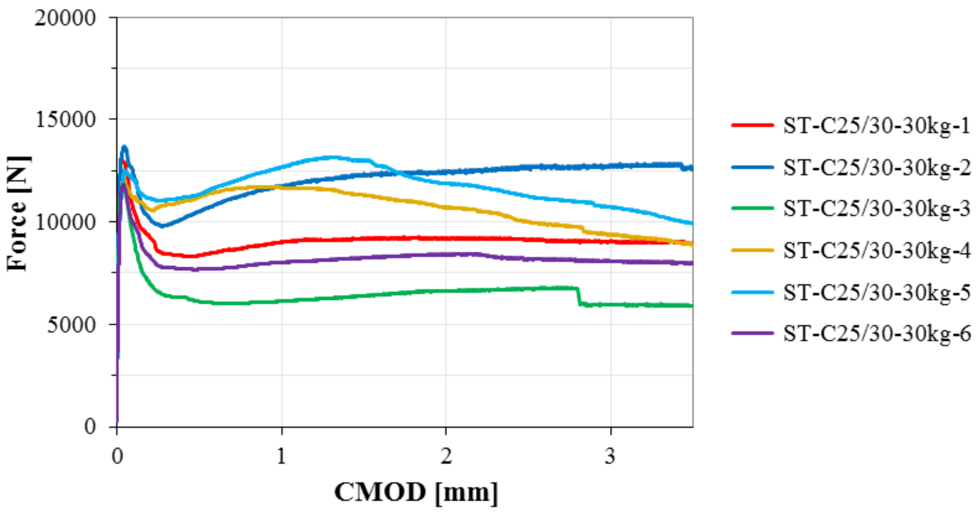

Fig. 2: High distribution of the results

4.2 The ideal distribution

The location of the fibre in the concrete could be characterized by its centre point and the angel between the global *x* and *y* axis, as its orientation. The perfect distribution in an infinite space is established when the distances among the centre points of the fibres are equal, and every fibre has a different orientation. The perfect distribution in a finite space is established likewise when the distance among the centre points are equal, and the fibres are oriented uniformly (uniform distribution of Φ and Θ, according to Fig. 3). In this situation the centre points of the fibres are located in a tetrahedral network, which side-length is defined by the dosage of the fibres. In this theoretical situation the average length of the fibres in the *x* direction could be calculated according to Romualdi and Mandel [12] with the aid of Fig. 3:

$$l_{\mathrm{f,x,m}} = \frac{\int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{2}} l_{\mathrm{f}} \cos\phi \cos\theta \, d\phi \, d\theta}{\frac{\pi}{2}\frac{\pi}{2}} = 0.405\, l_{\mathrm{f}} \qquad (1)$$

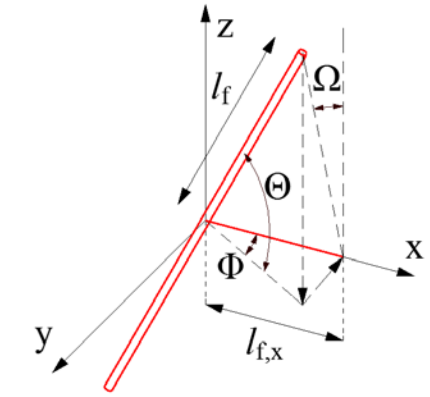

Fig. 3: Length of the fibre in the direction *x*

The orientation could be effected by the boundary of the formwork (wall-effect) and the average length could be modified (similar to [14]):

$$l_{\mathrm{f,x,m,tf}} = \frac{\int_0^{\frac{\pi}{2}}\int_0^{\Omega_{\mathrm{t}}} l_{\mathrm{f}} \cos\Omega \cos\theta \, d\Omega \, d\theta}{\frac{\pi}{2}\Omega_{\mathrm{t}}} \qquad (2)$$

where t_f is the distance of the middle point of the fibre from the formwork, according to Fig. 4.

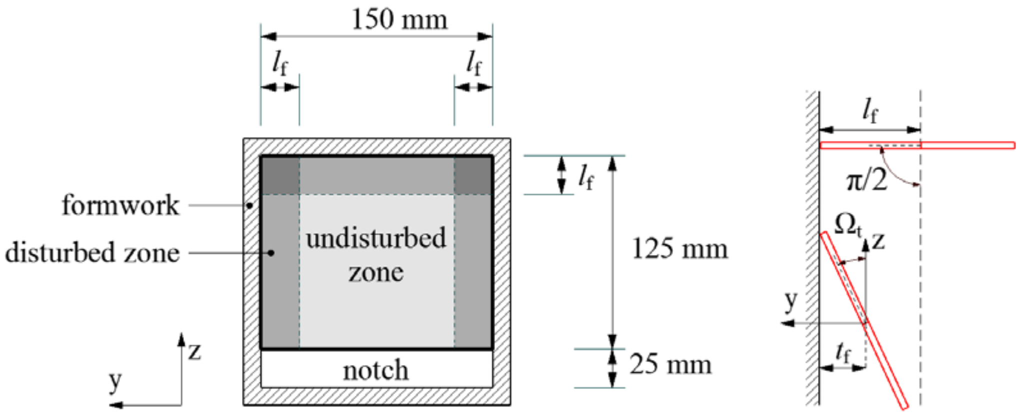

Fig. 4: Wall-effect of the formwork

By knowing the average length of the fibres the number of fibres at a unit cross section with the modification of the formula of Romualdi and Mandel [12] is then:

$$n_{\mathrm{a,u}} = \frac{2\, l_{\mathrm{f,x,m}}\, N}{V} \qquad \text{in undisturbed zone} \qquad (3)$$

$$n_{a,d} = \frac{2\, l_{f,x,m,tf}\; N}{V} \quad \text{in disturbed zone} \qquad (4)$$

Where n_a number of fibres crossing the section, pieces/mm^2

$l_{f,x,m}$ average fibre length in the undistributed zone, mm

$l_{f,x,m,tf}$ average fibre length in the distributed zone in a t_f distance from the formwork

N number of fibres in the volume

V volume, mm^3

Comparison of the number of fibres crossing the section by test results and by calculation (neglecting or calculating with the wall-effect) can be seen in Table 3. The results show that the method is relatively accurate, although the effect of the two-side formwork (in the corner) was not taken into account.

Tab.3: Comparison of the results of fibres per cross section, real and calculated values

Dosage [kg/m^3]	Fibres per cross section, pieces		
	Test results, average of 6 beams	Calculated results, without wall-effect	Calculated results, with wall-effect
20	31.8	24.3 (-23%)	30.2 (+1%)
25	38	30.4 (-20%)	37.8 (+1%)
30	51.8	36.5 (-30%)	45.3 (-12%)
40	71.9	48.7 (-32%)	60.5 (-15%)

4.3 Characterization of the real and ideal distribution

The fibres start to work after the crack of the matrix and they add ductility. The energy-absorbing function of the fibres could be according to Zollo [16]: 1) fibre bridging, 2) fibre pull-out, 3) fibre failure. Generally, steel fibres have fibre pull-out from the matrix [5] which could be modelled as a mostly permanent residual tension stress on the tension side of the section. This tension stress depends on the number of the fibres crossing the section. Knowing this number of the fibres a static-moment could be defined for the cross-section, called *fibre-moment* (S_f). To determine this value the fibres crossing the cracked section need to be analysed the following way. After a beam test was made the beam was severed to two parts and the cracked cross section was divided to five stripes with a thickness of 2.5 cm, parallel with the notch (Fig. 5). The number of fibres was counted in each strip. The compressed concrete zone compared to the size of the cross-section is relatively small and negligible, thus the distance between the compressed concrete part and the strip could be measured from the edge of the beam. The fibre-moment could be calculated as follows:

$$S_f = \sum_{i=1}^{5} t_i\, N_i \qquad (5)$$

Where S_f fibre-moment, mm

t_i distance of the strip from the edge of the beam, according to Fig. 5

N_i number of fibres in the actual strip

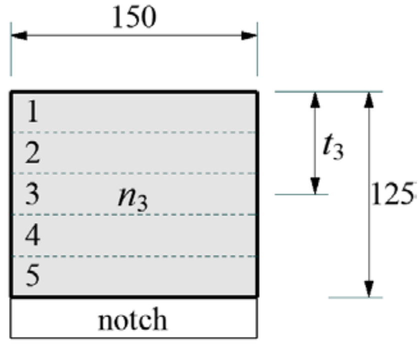

Fig. 5: Dividing the section

Taking into account the effect of the formworks and assuming the ideal distribution the *ideal fibre-moment* ($S_{f,id}$) could be calculated according to (6). To measure an ideal distributed fibre-moment without the effect of the formworks ($S_{f,id,u}$), i.e. material parameter for calculations, the (7) need to be used.

$$S_{f,id} = t_u\, A_u\, n_{a,u} + \int_{A_d} t_{d,i}\, n_{a,d,i}\, dA_d \tag{6}$$

Where
- $S_{f,id}$ ideal fibre-moment, mm
- t_u distance of the centre point of the undisturbed area, mm
- A_u undisturbed area of the cross section, mm^2
- $t_{,d}$ distance of the centre point of the disturbed area, mm
- A_d disturbed area, mm^2

$$S_{f,id,u} = \frac{125}{2} A n_{a,u} \tag{7}$$

Where
- $S_{f,id,u}$ ideal fibre-moment without the effect of formwork, mm
- A area of cross section (150x125 mm^2)

4.4 Work of the FRC

To measure the work of the FRC the area under the load-CMOD diagram could be used. This area could be divided to two parts: work of the plain concrete and added work of the fibres, *fibre-work* (W_f). To compare the beams the fibre-work was used as it is shown in Fig. 6.

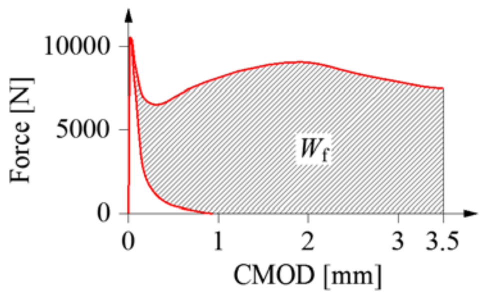

Fig. 6: Fibre-work

5. Relationship between the fibre-work and the fibre-moment

Fibre-work and fibre-moment relationships are shown by the diagrams in Fig. 7. The ideal fibre-moments with and without the effect of the formwork were also calculated and indicated in the diagrams. Lines were fitted to the data with linear regression, to the data with the same dosage (L_d) as well as the complete set (L). Similarly to the ideal fibre-moment an *ideal fibre-work* could be calculated ($W_{f,id}$). This could be done with the help of the L_d or the L. Five kinds of value were calculated for one type of sets (concrete class and dosage): mean value of the work of the fibre (W_m); ideal work of the fibre according to L_d, disturbed and undisturbed model ($W_{id,Ld}$, $W_{id,Ld,u}$); ideal work of fibre according to L also disturbed and undisturbed model ($W_{if,L}$, $W_{if,L,u}$). Differences between the values calculated by $S_{f,id}$ and $S_{f,id,u}$ were investigated, i.e. effect of the formworks. The results are shown in Fig. 7 by bar charts.

At the series ST-C25/30 the line fitted to the same dosage (L_d) and to the complete set (L) are different both in their slope and position. These lines must start from the origin, which physical meaning is that the fibre-work is zero at zero fibre-moment. At this series only the global line (L) satisfy this condition. The series ST-C30/37 and ST-C35/45 shows better results: most of the lines are parallel and close to each other. Only 30 kg dosage at ST-C30/37 has some anomaly. According to these results use of L instead of L_d is generally recommended.

It can be seen that the fibre-work calculated according to the L is almost in all cases smaller than the mean fibre-work. Only fibre-work calculated according to the L_d could be significantly bigger, but this is only at 25 kg at ST-C25/30. The differences between the calculation of disturbed and undisturbed model are between 8.8 to 12.4%. Material model used for elements avoiding the wall-effect could be calculated with the use of this reducing factor.

Even though the number of the fibres was the same at same dosage but different concrete class there is an apparent difference between the mean fibre-work and the calculated ones at the concrete class C35/45, comparing to the others. Although the global lines (L) of concrete classes are mostly overlapped, the high mean values must mean high fibre-moments. The reason for this could be find in the relationship of concrete mix, production of the beam and the dispersion.

6. Conclusion

The generally accepted testing method of fibre reinforced concrete is the three or four points bending beam tests, measuring the load and deflection and/or CMOD. These standard test methods could be easily performed in most of the laboratories, although the dispersion of the results could be high, even if everything is done in order to reach the perfect mixing. Generally, the measuring of the effect of the fibre is done according to concrete strength and fibre dosage. These test results include the wall-effect of the formwork, which increases the values. According to this research a parameter was defined, called fibre-moment, which could eliminate this effect. A new approach was presented where the effect of the fibres was calculated with the help of linear regression on the data set. This method could lead to reach more accurate data with less physical tests.

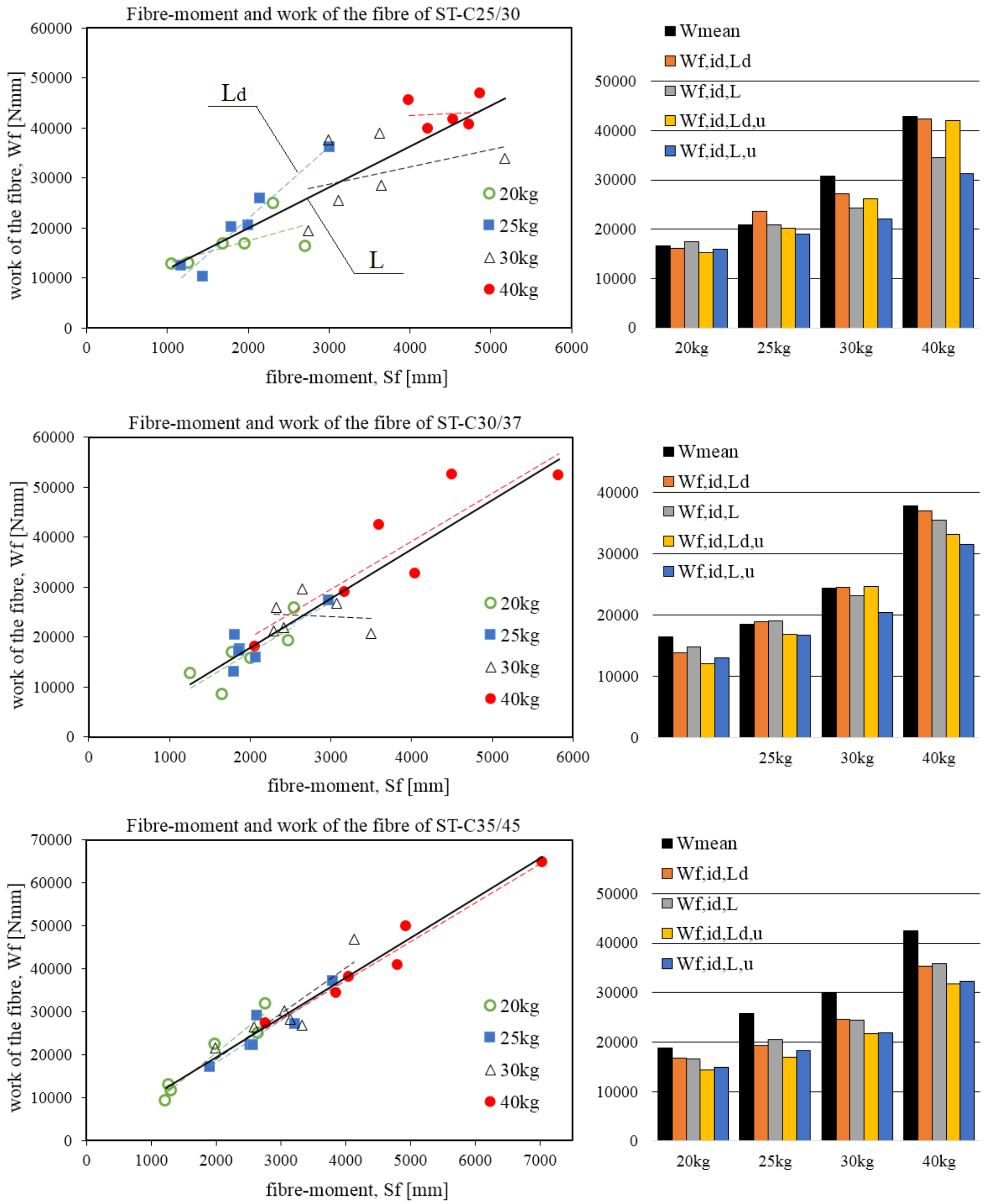

Fig. 7: Fibre-work and work of the fibre relationships with the evaluations

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