THE ISSUE OF NUMERICAL ANALYSIS OF FIBRE REINFORCED CONCRETE STRUCTURES

SAJDLOVÁ Tereza [1], PUKL Radomír [2], KABELE Petr [3]

Abstract

Utilization of the steel fibre reinforced concrete (SFRC) for certain types of civil engineering structures promises potential advantages in comparison to the traditionally reinforced concrete structures - efficient manufacturing, lower risk of corrosion, less damage during transport, etc. Since the design codes for SFRC structures are generally not available, computer simulation based on nonlinear finite element analysis has a big potential in investigation and design of these structures. Special material models accounting for the high toughness and ductility of the SFRC are recently available for realistic modelling of the SFRC-materials. Appropriate input material parameters can be identified from the measured response of the four-point bending beam tests using inverse analysis procedure. As an example the results from numerical simulations of SFRC layered structural elements are presented and the procedure of determining material parameters and model validation is shown.

Keywords: fibre reinforced concrete, numerical analysis, nonlinear material models, inverse analysis, layered structural elements

1. Introduction

Addition of steel fibres to the cementitious matrix is a promising method how to improve concrete behaviour mainly in tension. Steel fibre reinforced concrete (SFRC) has higher ductility, it can save amount of convention reinforcement, labour and in consequence costs of the structure. However, broader use of SFRC as construction material is limited among others by lack of design codes. Advanced methods as well-established nonlinear finite element analysis can help to design SFRC structures. Behaviour of the structure under service as well as ultimate conditions can be virtually simulated using computer methods quite realistically. Nonlinear fracture analysis accounts tensile capacity of material. In the case of SFRC it should be taken into account, that the tensile strength and especially the fracture energy of SFRC are substantially larger than in normal concrete.

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[1] SAJDLOVÁ Tereza, Department of Mechanics, Faculty of Civil Engineering, Czech Technical University in Prague, 166 29 Prague 6, Thakurova 7, Czech Republic, tereza.sajdlova@fsv.cvut.cz

[2] PUKL Radomír, Červenka Consulting, 150 00 Prague 5, Na Hřebenkách 55, Czech Republic, radomir.pukl@cervenka.cz

[3] KABELE Petr, Department of Mechanics, Faculty of Civil Engineering, Czech Technical University in Prague, 166 29 Prague 6, Thakurova 7, Czech Republic, petr.kabele@fsv.cvut.cz

In this paper, the typical procedure of modelling of SFRC structures is described. As an example, numerical analysis of SFRC layered structural elements is shown.

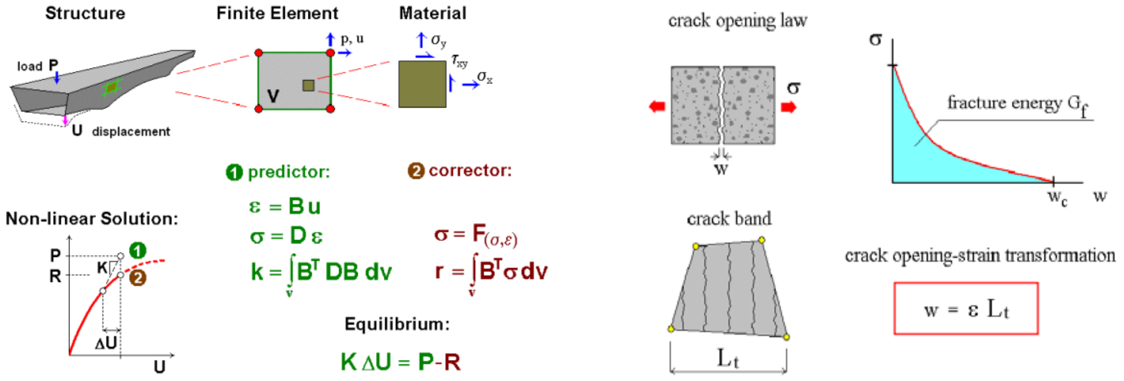
2. Modelling of SFRC Structures

Typical procedure of modelling a SFRC structure by nonlinear finite element analysis consists of several steps. The first one is the estimation of input material parameters based on mixture, contents and type of fibres, etc. The next step is modification of initial input parameters by inverse analysis of material tests, mainly four-point bending test, to better describe SFRC behaviour. The third and last step consist of analysis of whole structural element using determined material parameters.

2.1 Nonlinear Finite Element Analysis

The nonlinear numerical analysis of RC structures is based on finite element method and nonlinear material models for concrete, reinforcement and their interaction. Tensile behaviour of concrete is described by smeared cracks, crack band and fracture energy, compressive behaviour of concrete is described by damage model with hardening and softening. Reinforcement is usually modelled by truss elements embedded in two-dimensional isoparametric concrete elements. The nonlinear solution is performed incrementally with equilibrium iterations in each load step.

The finite element software ATENA is a proven tool for realistic computer simulation of damage and failure of concrete and reinforced concrete structures [1]. It contains advanced material models for concrete, reinforcement, bond etc. The tensile behaviour of concrete is modelled by nonlinear fracture mechanics combined with crack band method [2] and smeared crack concept, see Fig. 1. Principal material parameters for modelling of fracturing behaviour are tensile strength, fracture energy and shape of the stress-crack opening curve. For the normal concrete an exponential crack opening law derived by Hordijk [3] is applied.



(a) (b)

Fig. 1: (a) Scheme of the nonlinear finite element method, (b) Smeared crack model for tensile behaviour of concrete

In the smeared crack concept, the real discrete crack is substituted by a band of localized strains. The crack strain is related to the element size, which represents the crack band width L_t. Consequently, the softening law in terms of strains for the smeared model is calculated for each element individually, while the crack-opening law is preserved based on fracture energy consumption. This model is objective due to the energy formulation and its dependency on the finite element mesh is negligible.

The conventional reinforcement in standard reinforced concrete structures can be modelled as discrete bars or cables or as smeared reinforcement with given reinforcing ratio and direction. This kind of treatment of reinforcement is not well suited for description of fibres mixed in the concrete mass like in the FRC material. For this purpose special material models are introduced.

2.2 Material Models in ATENA Software

The tensile response of SFRC differs from normal concrete not only in the values like tensile strength and especially fracture energy, but also in the shape of tensile softening branch. The original exponential function valid for normal concrete can be used as a first approach, but preferably would be to use more realistic form of the tensile constitutive law. Therefore, special material models at macroscopic level are needed for modelling of SFRC-material in the numerical simulation of SFRC-based structures.

The most sophisticated and most general model of SFRC material represents an extension to the fracture-plastic constitutive law [4] called CC3-User model. It describes the tensile behaviour according to the material response measured in tests point-wise in terms of the stress-strain relationship. The first part of the diagram is the usual stress-strain constitutive law. After exceeding the localization strain ε_{loc} the material law assumed for the characteristic crack band width L_{ch} is adjusted to the actual crack band width L_t.

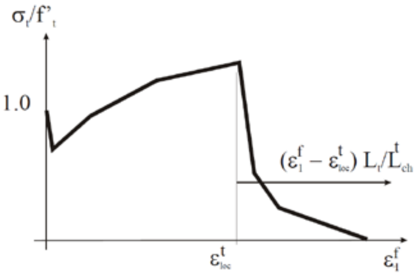


Fig. 2: User defined tensile behaviour

Compressive stress-strain law of mentioned material models is described in Fig. 3. The softening law in compression is linearly descending and the end point of the softening curve is defined by plastic displacement w_d (corresponding to ε_d in Fig. 3). By increasing material parameter w_d the contribution of the fibres to the compressive behaviour of concrete is considered. Another important compressive parameter for SFRC modelling is reduction of compressive strength due to cracks labelled as *RC* in the section 3 which says how the strength is reduced while the material is subjected to lateral tension.

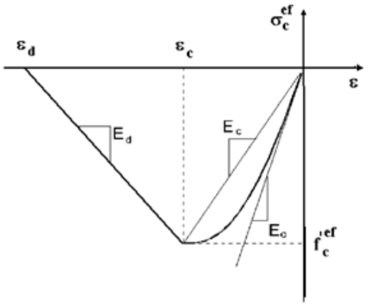


Fig. 3: Compressive stress-strain law

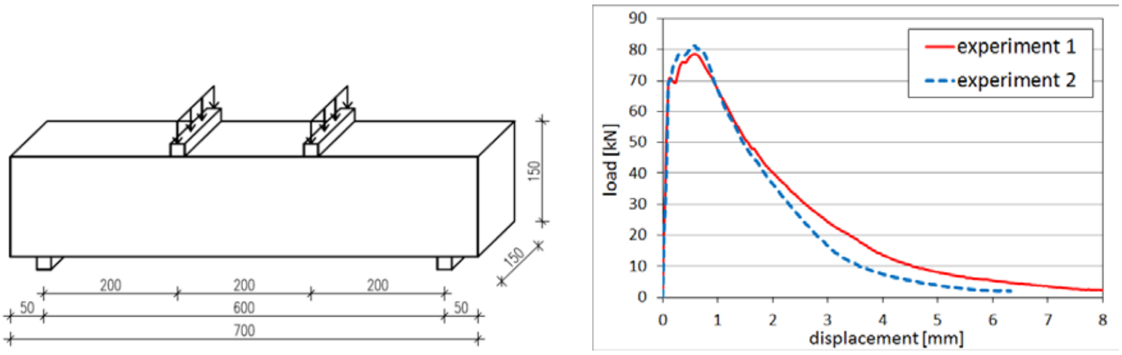
3. Application Example – Layered SFRC Structural Elements

As an example of procedure of SFRC modelling by nonlinear finite element method, layered SFRC elements were chosen. Layered SFRC elements are created from two or more layers of material with continuously variable composition and microstructure. Since the fibres are one of the most expensive components of the composite the aim is to use them as efficiently as possible and thereby reduce the amount of fibres and cost of materials at the same time. The principle consists in gradually changing the material composition and microstructure over a structural element volume so that the local properties meet the local load-resisting requirements.

The layered structural elements can be used for façade panels, floor structures in buildings (for new buildings as well as for reconstruction), for bridge structures or impact and blast resistant elements. In this paper, the layered element consisting of five layers and designed to sustain bending moment is presented.

3.1 Identification of SFRC Parameters

The SFRC input parameters must be determined for material corresponding to the class C110/130. Concrete matrix is reinforced with the steel fibres in volume fraction 1.5 %. Experimental verification of cylinder compressive strength after 28 days is 125 MPa, Young's modulus is 45 GPa. The identification of SFRC parameters is based on the results of four-point bending tests, see Fig. 4.

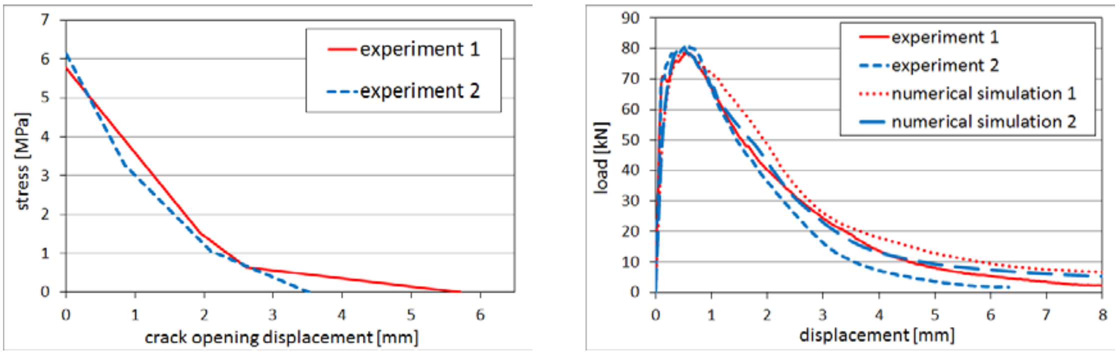


(a) (b)

Fig. 4: (a) Scheme of four-point bending test, (b) Experimental results

For the inverse analysis, stochastic approach can be used (randomization of initial material input parameters by probabilistic system SARA, numerical reproduction of four-point bending tests, comparison with experimental results and selection of appropriate parameters). Another approach is software Consoft developed by prof. Dr.-Ing. Volker Slowik and his colleagues at the University of Applied Sciences in Leipzig, Germany [5]. Automatic analysis based on the evolutionary algorithms is used for the determination of cohesive law function. The second approach was applied in this study.

The best results of inverse analysis are shown in Fig. 5. Softening curves for two material models are shown in the Fig. 5 (a), comparison between numerical and experimental results is shown in the Fig. 5 (b). The results of numerical simulations are in a perfect accordance with the experimental results.

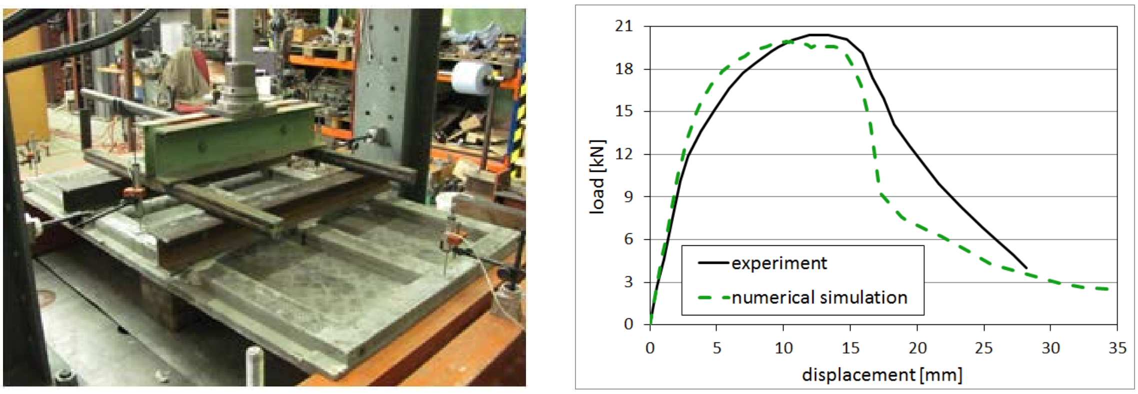


(a) (b)

Fig. 5: (a) Softening curves, (b) Comparison of experimental results and numerical simulations of four-point bending tests

3.2 Validation of modified material model

Material model 3D NLC2 User with softening curve derived in the previous chapter was validated by the numerical simulation of bridge slabs used for reconstruction of a bridge in Czech Republic; they should serve as a permanent formwork [6]. Slabs are made from the same material as was described in the previous chapter. Four-point bending tests of these slabs were carried out as shown in Fig. 6 (a) – photo from the lab of the Klokner Institute of CTU. The dimensions of the slab are 1 x 1.67 m, the thickness is 20 mm, the rib thickness is 60 mm and thickness of the central rib is 40 mm. Load bearing capacity of the model was 20.6 kN which is pretty close to the experiment. The failure (fracture) of the slab occurred also in accordance with the experiment in the transverse direction approximately at the edge of loading plate.



(a) (b)

Fig. 6: (a) Four-point bending test of slab, (b) Comparison of experimental and numerical results

3.3 Numerical Simulation of Layered SFRC Element

Layered beam designed to sustain bending moment consists of five layers whose fibre volume fraction increases linearly from the upper surface toward the bottom. Fibre volume fractions of all layers are depicted in Fig. 7. Total volume fraction is identical with the homogenous beam from the previous chapter, i.e. 1.5 %. Based on the cohesive law for fibre volume fraction 1.5 % derived by inverse analysis, softening curves for other layers are determined by micromechanical model presented in [7], see Fig. 8 (a).

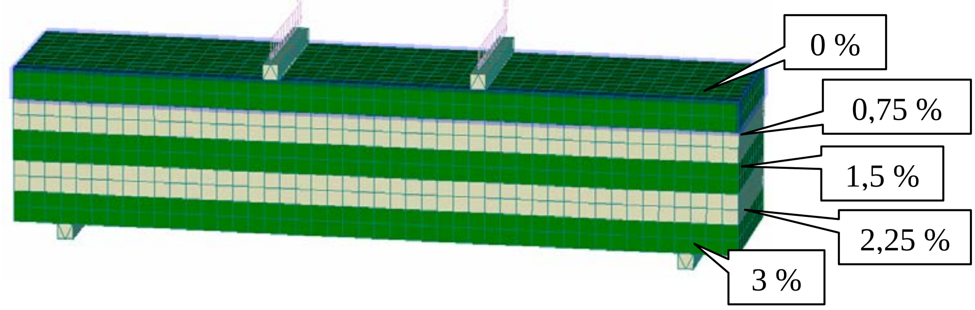
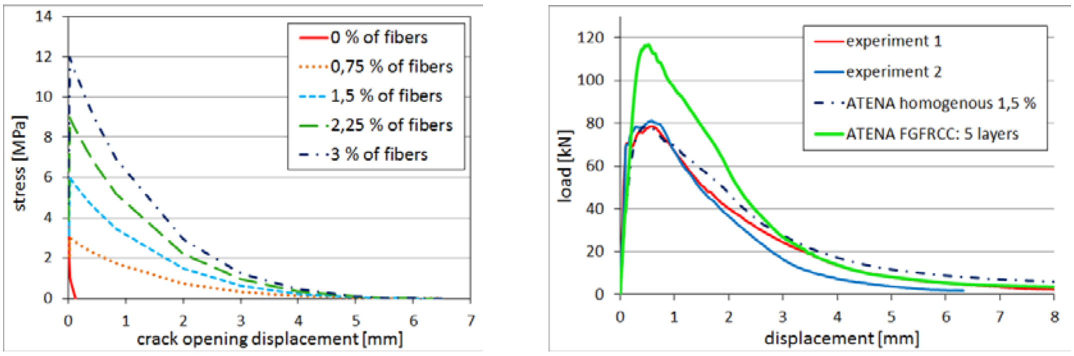


Fig. 7: Layered structural element – fibre volume fraction of layers

The spatial variability of mechanical properties was modelled by division of the structural member into macroelements representing different layers of layered member. Each macroelement has specific material model with different fibre volume fraction. Macroelements are connected with each other by an interface with the same properties as the material of macroelements.



(a) (b)

Fig. 8: (a) Softening curves for different fibre volume fraction, (b) Comparison of layered and homogeneous beams

Load-displacement diagram of the layered model is shown in Fig. 8 (b) in comparison to the homogenous beam with fibre volume fraction of 1.5 %. The comparison of the curves shows that the layered beam achieves 46% higher load bearing capacity (117 kN compared to 80 kN). Comparison of crack width in the results of numerical analysis is also an important criterion for quality evaluation - at the load of 80 kN maximum crack width in homogenous beam is 0.25 mm while for layered beam it is only 0.012 mm, see Fig. 9.

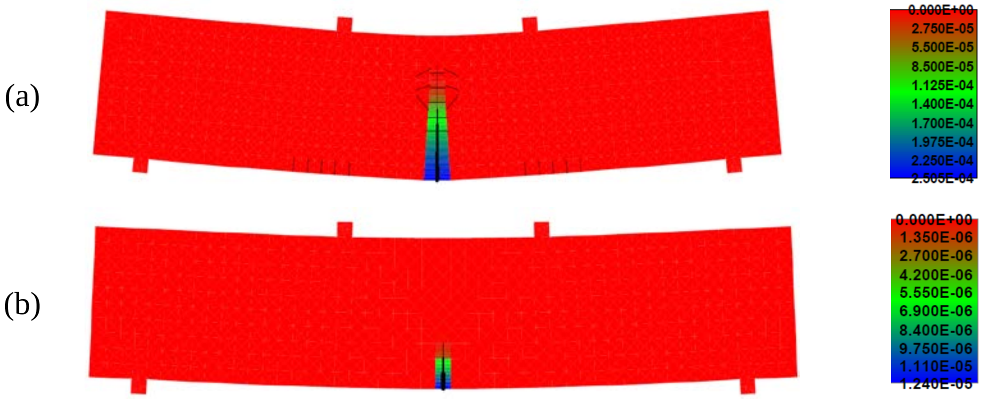


Fig. 9: Cracks in beam for load 80 kN – (a) Homogeneous beam, (b) Layered beam

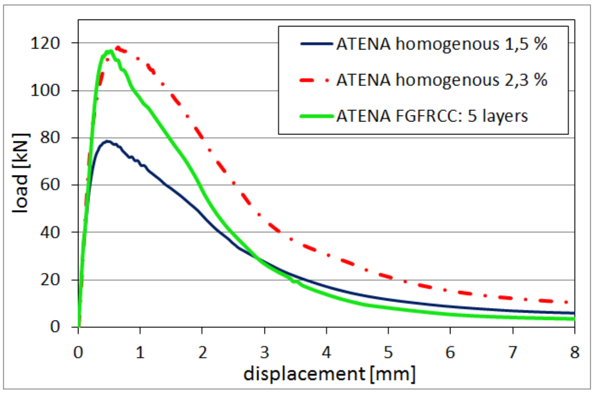


Fig. 10: Comparison of homogeneous and layered beams

The results show that the proper arrangement of layers can increase the load bearing capacity by 45 % at the same total quantity of steel fibres. In this specific case, the load bearing capacity of layered element (with a total fibre volume fraction of 1.5 %) is comparable with the load bearing capacity of homogenous element with 2.3 % fibres by volume, see Fig. 10. By the nonlinear numerical analysis is shown that it is possible to save 35 % of fibres and still achieve the same load bearing capacity.

4. Conclusions

The nonlinear finite element modelling can be successfully used for analysis of behaviour and failure of SFRC structures. Crack initiation and development, load carrying capacity and post-critical behaviour of the structures, structural parts or experimental specimens can be investigated. Advanced material models for numerical simulation of fibre reinforced concrete are available. Determination of appropriate material parameters for model is of crucial importance. As shown in the paper, the required values can be efficiently determined using inverse analysis of basic experiments as four-point bending test.

The successful procedure of determining the input parameters, validation of the parameters and finally prediction of the structural behaviour using determined data is shown on SFRC layered elements that are the object of authors´ research project.

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