LOAD BEARING CAPACITY OF STONE ARCHES OF LEGION BRIDGE

Marek Vokál, *

Department of Concrete and Masonry Structures, Faculty of Civil Engineering,
Czech Technical University in Prague, Thakurova 7/2077, 166 29 Prague 6, Czech Republic.
marek.vokal@fsv.cvut.cz

ABSTRACT

The article deals with available engineering assessment methods for the stone vault bridges according to relevant standards. . The article summarizes variable methodologies used for design and assessment of masonry vaults in last period. All methods were applied for assessment of the Legion Bridge over Vltava River in Prague and results were compared.

KEYWORDS

Masonry arch bridge • Vault • Bridge • Assessment • load-carrying capacity • Thrust line

1. INTRODUCTION

When assessing existing bridges, the requirement for sufficient mechanical resistance and stability is usually given by the maximum load that the structure is able to carry safely - the maximum weight of the road vehicle which can pass the bridge under the specified conditions, i.e. the load carrying capacity. Determination of the load bearing capacity of the existing bridge is governed by the same principles as the design of new structure (see applicable technical standards and regulations EN, DIN, and MVL). However, some material and load factors are of different values. In case of masonry vault structures this task is complicated by the structural behaviour and typical property of the material - a negligible tensile strength, see (Vokál 2017). Due to these facts, the procedures for determination of the load carrying capacity are non-linear and must involve a large number of parameters with significant variability. Therefore, it is very difficult to set up a simple analytical model and special programs developed directly for vault structures are usually used.

In case of road bridges three kinds of load carrying capacity (according to (Drahorád 2013*b*)) can be calculated :

- Vn - normal load carrying capacity of the bridge – represents the maximum weight of one typical lorry, which may pass the bridge without any restrictions (position limited by safety barriers only)
- Vr - exclusive load carrying capacity – represents the maximum weight of one truck, which can pass the bridge as single vehicle in any position or lane respectively (no other traffic loads except pedestrians is permitted). It can move anywhere on the bridge.
- Ve - exceptional load carrying capacity - which represents the maximum weight of special vehicle, which can pass the bridge under special conditions (specified velocity, specified path and eccentricity) .

* Supervisor: Prof. Ing. Alena Kohoutková, CSc.

Determination of each load carrying capacity presents separate calculation in both limit states (SLS and ULS) considering all other relevant loads (temperature, wind, flood etc.).

2. BRIEF HISTORY AND DESCRIPTION OF THE BRIDGE

The Legion bridge connects the Old town with the Lesser town of Prague through the Střelecký Island. The foregoer bridge of Legion Bridge was Bridge of Emperor František and it was second bridge in Prague finished in 1841. The Legion Bridge was constructed at the position of Bridge of Emperor František. The construction of new bridge ran from 1898 to 1901. In contrast to the original bridge, the superstructure is made of a massive stone vault structure. The construction of the bridge consists of nine flat vaults of different spans: 26.6 + 34.3 + 38.5 + 42.0 + 27.8 + 27.8 + 31.9 + 28.7 + 25.6 m. Two vaults above the Střelecký Island are vaults of circular segments, the other vaults are elliptical. The construction of the bridge is made of granite blocks with a gap of 12 - 15 mm filled with cement mortar. The stone facade of front walls of light sandstone and red granite symbolizing national colors. The bridge has wide footways and motorway lanes, electric tracks were also built on the bridge, trams run along an existing bridge from June 17, 1901.

The Legion Bridge is an immovable cultural monument and is therefore protected according to the provisions of law On State Monument Care, as amended. Due to the fact that it is a building located in the territory of the Prague Historical Reserve (PPR), the provisions of the Government Decree On the Historical Heritage in the Capital City of Prague. The historical monument in Prague, representing the historical center of Prague, was included in the UNESCO World Heritage List in 1992.

3. LIMIT STATES

3.1. Ultimate limit state

In the ultimate limit state (ULS) the behaviour of the structure just before the collapse is investigated. For the bearing capacity determination load factors according to appropriate EN are considered (1.35 for dead load and 1.35 for live load). Generally, it is assumed that plastic hinges are fully developed through the structure.. For details on masonry arch bridges behaviour see in (Drahorád 2013*a*) and (Vokál 2018). Resistances for axial and shear forces at the ULS can be written according to (Pume 2005) as:

$$N_{Rd} = f_d b \left(h - 2e_u\right) \tag{1}$$

$$V_{Rd} = \left(f_{vk0} + 0.4\sigma_d\right) b \left(h - 2e_u\right) / \gamma_M . \tag{2}$$

where: f_d is the design strength of masonry in compression, f_{vk0} is the characteristic value of initial shear strength at normal stress equal to 0, b, h is width or height - respectively, e_u is the eccentricity of the resultant axial force in cross-section at the ultimate limit state, σ_d is the design compressive stress in the compressed area at the ultimate limit state (uniformly distributed, see figure 1, m is the coefficient of friction in the masonry joint, γ_M is the factor of the material.

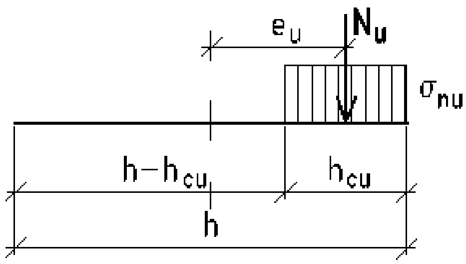

Figure 1: *Stress distribution of masonry cross-section at the ultimate limit state.*

3.2. Serviceability limit state

The serviceability limit state (SLS) describes the behaviour of the structure under ordinary operating conditions. Fulfilling the conditions of serviceability limit state provides the required properties and behaviour of the structure throughout its lifetime. In terms of serviceability limit state, crack width and structural stress under operating load are verified (load factor equals to 1.0). In terms of verification of vault structures, it is necessary to verify the maximum axial stress in the cross-section and the height of the compressed area at the cross-section (see (Hrdoušek 2008) and (Pume 2005). Elastic behaviour of the structure is considered with a linear distribution of axial stress in the compressed area of the cross section. Tensioned part of the section is excluded for stress determination (see figure 2).

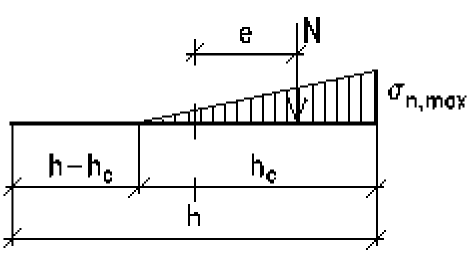

Figure 2: *Stress distribution of masonry cross-section at the serviceability limit state.*

$$\sigma_{n,max} = \frac{N_{Ek}}{3b\,(h-2e)} \leq 0.45 f_k \tag{3}$$

$$h_c \geq \frac{h}{2} \longrightarrow e \leq \frac{h}{3} \tag{4}$$

$$e = \frac{M_{Ek}}{N_{Ek}} \tag{5}$$

where: M_{Ek}, is characteristic moment caused by load, N_{Ek} is characteristic normal force caused by load.

4. METHODS OF CALCULATION - ARCH

1. Graphical method (controls all the requirements)
2. Linear calculation (controls all the requirements)
   (a) Beams – 2D or 3D
   (b) Plane-stress elements – 2D
   (c) 3D solid elements
3. Non-linear calculation (controls SLS requirements)
   (a) 2D – plane
   (b) 3D – solid elements – not used in this article
4. Equilibrium method — LimitState:Ring (controls only collapse of the structure)

4.1. Graphical methods

Various graphical methods had been used until computer aided design came to engineering practice. It provides very simple and quick design approach independent on arch bridge shape. Graphical methods are based on the thrust line determination. Thrust force at the cross section can be found as centroid of the axial stress diagram. When the thrust line is known, stress at the cross-section can be calculated as well. The method of finding the thrust line runs in following order (symmetric arch according to (Lipanská 1998)):

1. Divide the arch in 2 symmetric parts, find the weight of one half and from the geometry of arch we find force H (horizontal force in the top of arch)
2. Divide one half of arch into several partitions (vertical lines can be used to divide)
3. Draw graphical representation of all parts – Fi - size of vector in chosen scale, acts in its centroid
4. Force H we locate for example in the upper bound of cross section core and reaction in the lower bound of cross section
5. For getting the resulting force R1 in first partition, we graphically add the force Fi to H, for getting next resulting forces in each partition, we graphically add the forces Fi to previous resulting force

Basic principle of calculation is shown on Fig.3 and (Vokáč 2018).

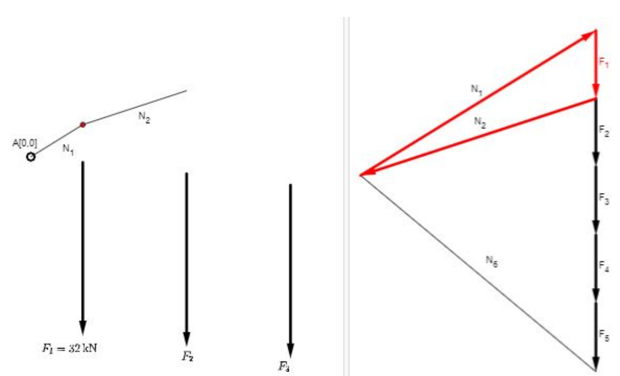

Figure 3: *Basic principle of the graphical method.*

4.2. Linear calculation

Structural analysis using linear calculation method was done in two different options. In the first one the arch was represented by sequence of beams in its center line. The backfill was represented by vertical beams provided in appropriate longitudinal distance, see figure 4. While modelling using linear calculation, one should not forget, that this calculation method doesn't take into account material non-linearity (and geometry changes due to excluding the tensioned part of cross section). For the second option the arch and backfill were modelled by the 3D solid elements with various mechanical properties – see figure 9.

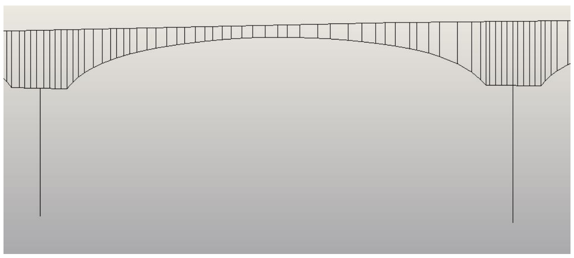

Figure 4: *Linear 2D beam model.*

For the second option the arch and backfill were modelled by the 3D solid elements with various mechanical properties – see figure 9.

4.3. Non-linear calculation

For performing an non-linear analysis and assessment of the structure, the mate- rial – mortar and masonry elements - is homogenized to preserve its properties in relation to the real behaviour of the structure or its part. It is assumed that the dimensions (thick) of the masonry elements and joints between them do not significantly affect the distribution of stress in the masonry element. The real stress-strain diagram of the masonry shows non-linear behaviour (see (Vokál 2017)) particularly due to negligible tensile strength. In this article, it is considered that the material acts only in compression and when the tensile stress occurs, cracks open up, see figure 5). If, subsequently, (e.g. in another load combination) the tensile stresses in the cross section disappear, the cracks close and the cross-section acts again as full.

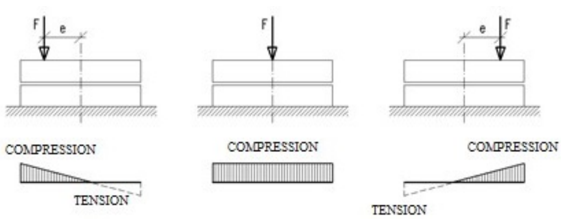

Figure 5: *Non-linear behaviour of masonry.*

The structural model in program Midas was prepared using plane-stress elements, modelling the joints between the granite blocks as set of elastic links with the property "Compression only", see figure 6.

4.4. Equilibrium method on rigid blocks

LimitState:RING is a special analysis software for checking the load bearing capacity of the vault in the plane of the longitudinal

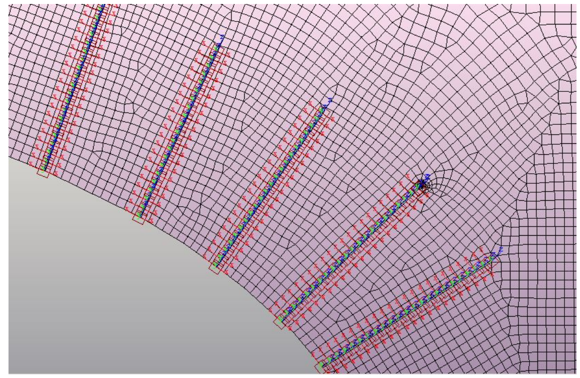

Figure 6: *Elastic links between nodes in joints of masonry.*

section of the bridge structure, including the load distribution by the backfill. It uses equilibrium equations on the parts of vault act as a rigid bodies, the structure is divided into rigid bodies depending on forming of the plastic hinges in locations with the lowest height of compressed area. Load distribution is considered according to Bousinesq, see example in figure 7. For more details see (Drahorád 2015).

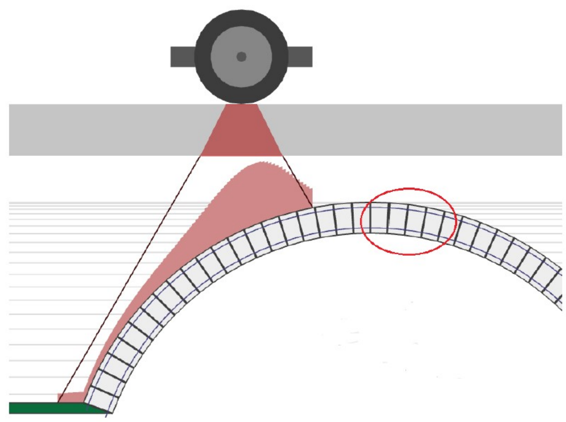

Figure 7: *Vertical traffic load distribution (dispersion) to the vault considered for arch modelling in LimitState:RING software.*

5. METHODS OF CALCULATION - TRANSVERSE DIRECTION

5.1. Linear calculation

5.1.1. Beam elements

Modelling using this method is done by representing the arch by beams in its middle line and dividing the backfill in chosen interval to represent it by beams as well, see figure 4. If the model shown in figure 4 is copied several times in the transverse direction the analysis model representing the arch as a body can be arranged. The stiffness of transverse beams is chosen as a stiffness of arch. Such 3D linear model can be used to study the effect of eccentricity of live load on the bending moments distribution in the transverse direction – see figure 8.

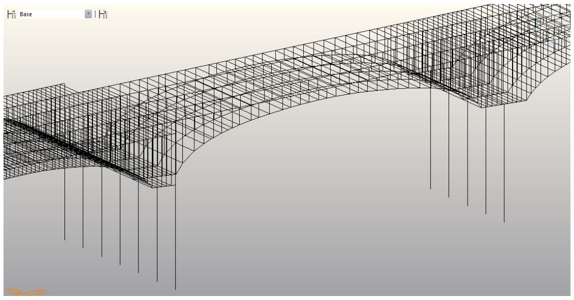

Figure 8: *Linear 3D beam model.*

5.1.2. Solid elements

Another option for linear analysis is to model the body of the structure by the 3D solid elements – see figure 9. Both longitudinal and transverse direction can be modelled by this way.

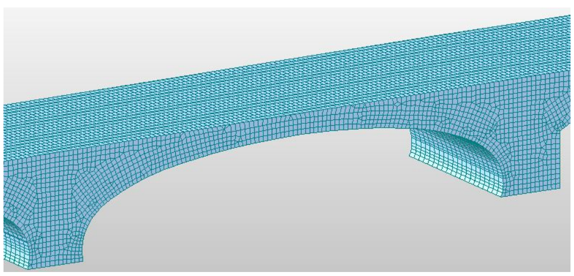

Figure 9: *3D solid model.*

5.2. Effective width

Principles of "modelling" of the bridge span 4 in transverse direction using effective width can be seen from figure 10. This way of modelling is used in most codes. It considers conservative idea, that non-loaded lane of arch doesn't carry any load. So the shear and bending stiffness between loaded and non-loaded elements is considered equal zero.

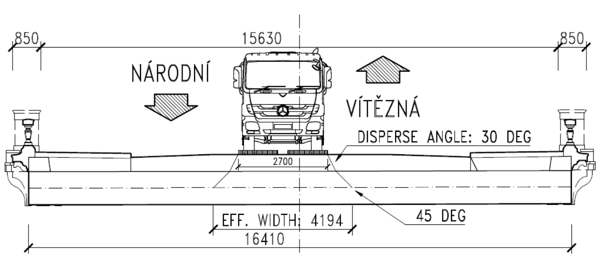

Figure 10: *Calculation of effective width.*

6. RESULTS

6.1. Graphical method

Result of graphical method for span 4 are shown in figure 11.

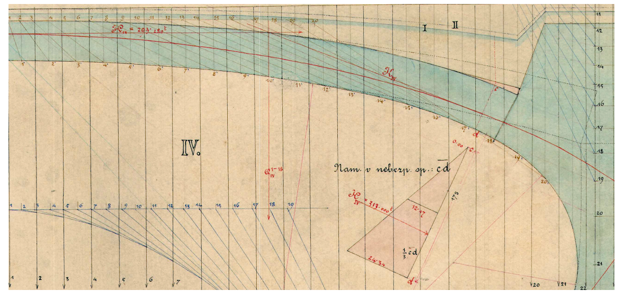

Figure 11: *Graphical solution of the span 4 from the archive documentation.*

6.2. Linear calculation

The bending moments on the beams are shown in figure 12.

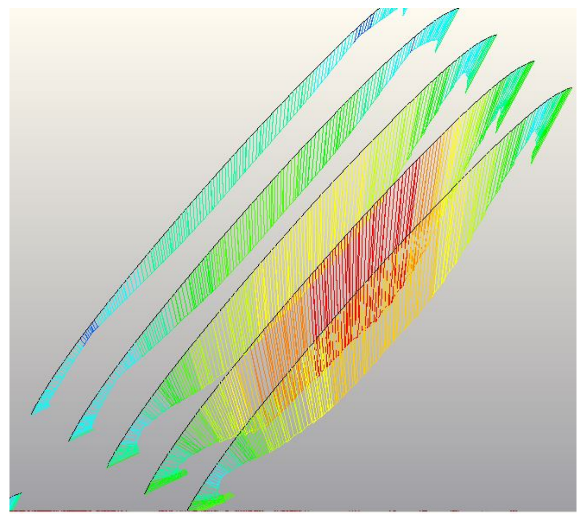

Figure 12: *Bending moment on the beams representing arch in longitudinal direction.*

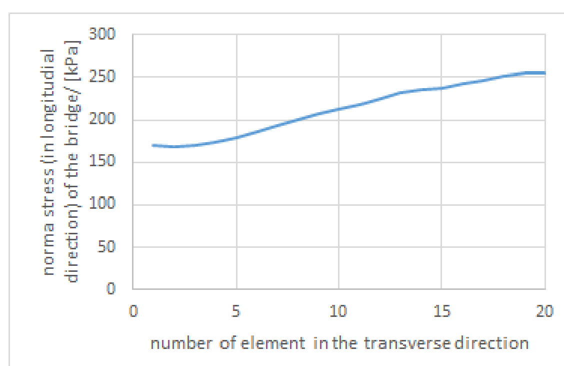

Figure 13: *Resulting stress from the load of exclusive load model.*

6.3. Non-linear calculation

Principal stress in the arch of span 4 can be seen in figure 14.

The legend of curves in the figure 15 is following:

sw means self weight

N means non-linear combination

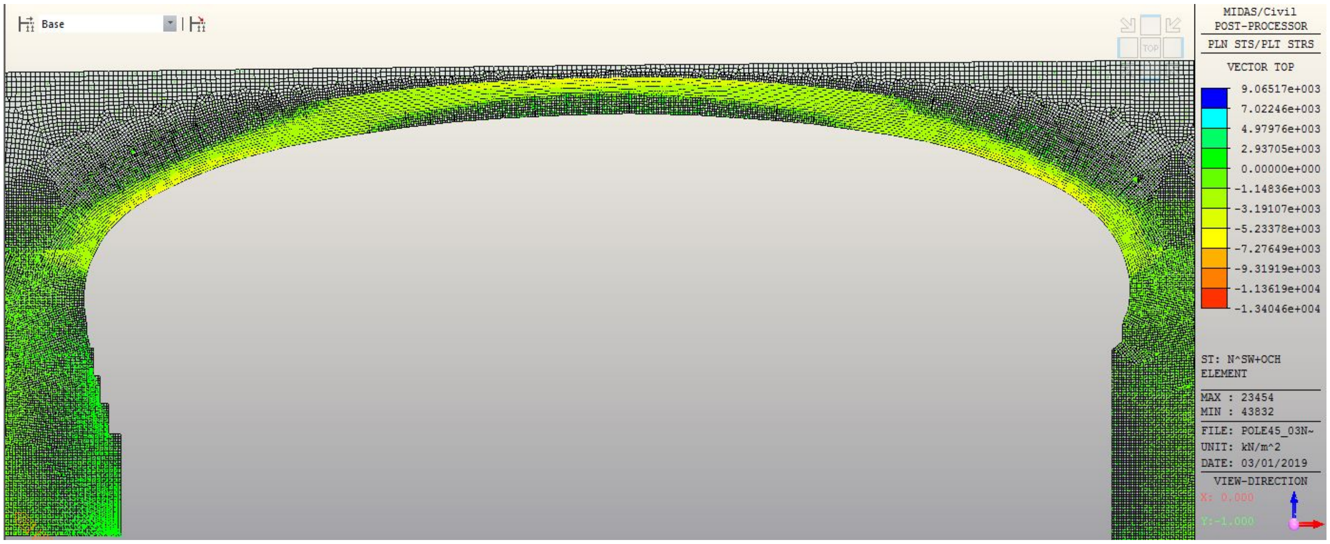

Figure 14: *Principal stress from the non-linear model.*

ohr means heat-up

och means cooling

$4VnT!$ means load by live load

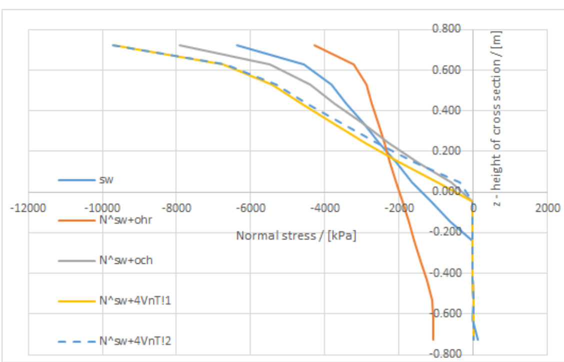

Figure 15: *Normal stress distribution in the middle of span 4 versus the cross section height.*

7. COMPARISON OF METHODS AND DISCUSSION

7.1. Self weight – the main load

In the figure 16 we can see the eccentricity from the linear, non-linear model and from graphical solution from the archive documentation. It is evident, that eccentricity from the non-linear model is higher, as expected, because tension, which is allowed in the linear model pushes the resultant thrust line to the centroid of the cross section. The non-linear analysis is more time consuming (on the effort of the engineer as well as the effort of the computer), but the real behaviour of the structure is better described by the non-linear model (see the behaviour of masonry in figure 5). The results from the non-linear model are more dangerous and are closer to the limit states.

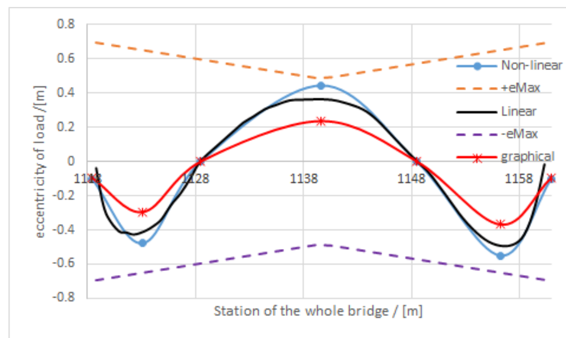

Figure 16: *Comparison of linear, non-linear and graphical method for span 4.*

7.2. Traffic load and its distribution in transverse direction

Load distribution in the transverse direction was compared on three models – 3D solid, 3D beam model and effective width model. Result can be found in figures 12, 13 10. 3D linear solid model assumes linear behaviour in all directions, therefore gives the most non-conservative results. The most conservative model is effective width model, because it entirely excludes part of cross section. The real behaviour is somewhere between. The beam model gives results between the two mentioned method, in opinion author is therefore the most real. The beam model is much simpler and the stiffness in transverse direction can be easily changed. Real 3D solid model (which considers non-linear behaviour) is complicated with the fact, that we don't know many crucial characteristics of masonry – such as bending and shear stiffness of mortar between the blocks – neither in longitudinal nor the transverse direction. That is the reason the 3D solid model is recommended just for special structures and if we know the parameters.

7.3. Final results of load carrying capacity:

The final results of load carrying capacity reflects both the results of modelling the arch itself and modelling of transverse direction.

It is in general known, that LimitState:RING gives non-conservative results. Non-linear model gives conservative results, first because

Table 1: *Resulting load carrying capacity*

Arch model	Transverse model	Vn	Vr	Ve
Linear	3D beam	41	122	230
Non-linear	effective width	32	83	185
LimitState:RING	effective width	46	105	182

of the method of assessing the load distribution in transverse direction, second because of modelling of the arch itself. The linear model results are between two mentioned method. For Ve and Vr, the beam model gives the most non-conservative results. The load is concentrated to small strip of the arch in effective width model, but in 3D beam model all the beams carry part of the load.

8. CONCLUSION

Several models of the Legion bridge were carried out. In opinion of the author, the most real behaviour of arches describes the non-linear model, because it considers the non-linearity, which impacts the calculation the most – negligible strength in tension. The results from modelling are non-conservative in comparison to other methods. Modelling of load distribution in transverse direction showed, that 3D solid model gives upper bound (non-conservative), effective width gives the lower bound (conservative results) and 3D beam model is somewhere between, which is the the most real behaviour. However, using effective width is precise enough for small spans (spans of 90 % of stone arch bridges are lower than 10 m), but leads to very non-conservative results for such a bridge with very large span (span of Legion bridge is the largest in Czech Republic). For modelling of such a large spans therefore other 3D models should be made.

ACKNOWLEDGEMENTS

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References

Drahorád, M. (2013*a*), 'Load–bearing capacity of masonry arch bridges', *International Conference Engineering Mechanics* **19**(1), 33–34.

Drahorád, M. (2013*b*), *ČSN 73 6222 – Load bearing capacity of road bridges*, ÚNMZ.

Drahorád, M. (2015), *The guide to use of software LimitState: Ring for employees of SŽDC*, Drahorád, M.

Hrdoušek, V., D. M. (2008), *ČSN P 73 6213 – Design of masonry road bridges*, ÚNMZ.

Lipanská, E. (1998), *Historic vaults*, El Consult.

Pume, D. (2005), *EN 1996–1–1 – Design of masonry structures*, ÚNMZ.

Vokáč, M. (2018), *The graphical solution of thrust line of a vault*, Česká technika.

Vokál, M. (2017), Non-linear analysis of slender masonry column, *in* 'PdD workshop', Vol. 8, pp. 48–50.

Vokál, M., D. M. (2018), 'The load bearing capaciy of railway masonry arch bridges', *Transactions of the VSB-Technical university of Ostrava* **18**(2), 12–18.