NUMERICAL MODELLING OF CONCRETE STRUCTURES EXPOSED TO RADIATION

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ABSTRAKT

Práci lze rozdělit do dvou částí zaměřujících se na dva přístupy k numerické analýze betonových konstrukcí vystavených ionizujícímu záření. První část využívá k analýze metodu konečných prvků. Druhá část pak diskrétní model, konkrétně Rigid-body-spring model.

Část MKP prezentuje postup vývoje programu pro analýzu betonového stínícího prstence ve 3D, diskutuje volbu modelu dotvarování takové konstrukce a poukazuje na možnost přesnějšího vystižení skutečného zatížení vlivem neutronového záření.

Druhá část nastiňuje postup práce na tvorbě diskrétního modelu ve 3D. Jsou uvedeny ukázky počátků vývoje programu, složitost diskretizace oblasti do Voroného diagramu a jsou diskutovány různé přístupy výpočtu a jeho předpokladů.

KLÍČOVÁ SLOVA

Numerická analýza; Beton vystavený záření; Metoda konečných prvků; Modely dotvarování; Rigid-body-spring model; Voroného diagram ve 3D prostoru

ABSTRACT

This work can be divided into two parts focusing on two approaches to the numerical analysis of concrete exposed to ionizing radiation. The first part uses the finite element method for the analysis. The second part uses a discrete model, namely the Rigid-body-spring model.

The FEM part presents the procedure for developing a program for the analysis of a concrete shielding ring in 3D, discusses the choice of a creep model for such a structure, and points out the possibility of a more accurate representation of the actual loading due to neutron radiation.

The second part outlines the workflow for creating a discrete model in 3D. Examples of the early development of the program are given, complexity of discretizing a region into a Voronoi diagram is discussed along with various approaches to calculation and its assumptions.

KEYWORDS

Numerical analysis; Concrete exposed to radiation; Finite element analysis; Concrete creep modeling; Rigid-body-spring discrete model • Voronoi diagram in 3D space

1. INTRODUCTION

This work can be distinguished into two parts – first part shows the progress in development of 3D FEM (following continuous work introduced in Kovář (2023), (Kovář (2024)), used for the analysis of a concrete biological shield (CBS). Mainly, the creep behaviour was added simulated by fib Model Code 2010 creep model. However, better suitability of the B3 model is discussed. In addition, a model for validation of the results obtained by the custom-made software was made in a verified commercial software (ATENA). This model is presented in the next chapter.

The second part of this paper focuses on using discrete models, namely Rigid-body-spring model and its extension to 3D (again continuing the development introduced earlier in Kovář (2024)). This paper discusses challenges in propagating into 3D – the discretization into Voronoi tessellation, connecting the discrete cells and defining the constitutive laws between them.

2. 3D FINITE ELEMENT MODEL

Previous work (Kovář (2023), Kovář (2024)) described the ongoing development of custom software for finite element (FE) analysis. This software has since been further improved, specifically by the initial implementation of a creep behavior simulated by the creep model according to the Model Code 2010.

However, during the implementation process, several limitations of the Model Code 2010 model became apparent, particularly the fact that many of its parameters are predefined for standard structural elements and typical design situations. As a result, its applicability to more complex or non-standard structures is limited. To address this, implementation of the B3 creep model, which offers a more generalized and flexible formulation, is planned instead. The B3 model supports a wider range of material properties and loading histories, making it more suitable for detailed research applications. This is especially relevant in the context of the present study, where long-term non-typical loading effects are key considerations. Furthermore, the B3 model is empirically well supported and has been shown in multiple studies to provide more accurate results than the Model Code 2010 model, particularly for long-term deflection and creep prediction. This is supported by several comparative studies (Al-Manaseer & Prado (2015), Al-Manaseer & Lam (2005)).

In parallel with the software development, a verification model was constructed using a commercial, well-established, and validated finite element program (ATENA). This model serves as a reliable benchmark for validating the results produced by the in-house software. The following chapter provides a brief description of the initial verification model, presents the obtained results, compares them with the previous outputs of

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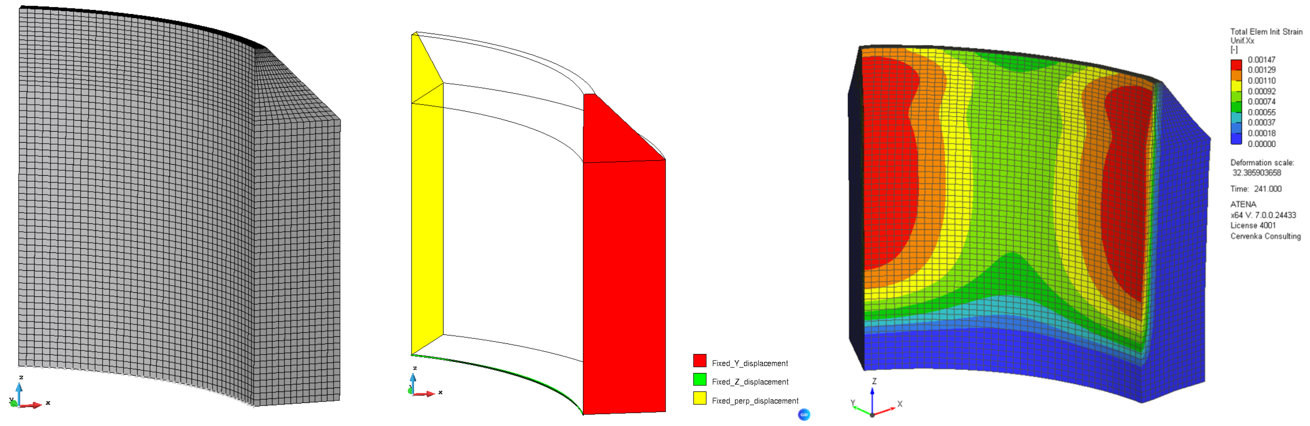


(a) *FE mesh* (b) *Support conditions* (c) *Initial strain distribution*

Figure 1: *Verification 3D model: (a) FE mesh, (b) support conditions, (c) initial strain distribution due to RIVE.*

the custom-developed software, and discusses the similarities and differences.

2.1. Description of the 3D Verification Model

The verification model (Fig. 1) respects the original geometry of the structure. A mesh composed of hexahedral elements was used with element size mostly 50 × 50 × 75 mm (see Fig. 1a). The concrete was assumed to be of higher quality, class C 50/60, with a density of 4 t/m^3.

Supports are defined as shown in Fig. 1b. The bottom inner edge is restrained in the Z-direction. The nodes on the faces where the structure continues are restricted in displacement perpendicular to the face, to reflect the symmetry of the structure.

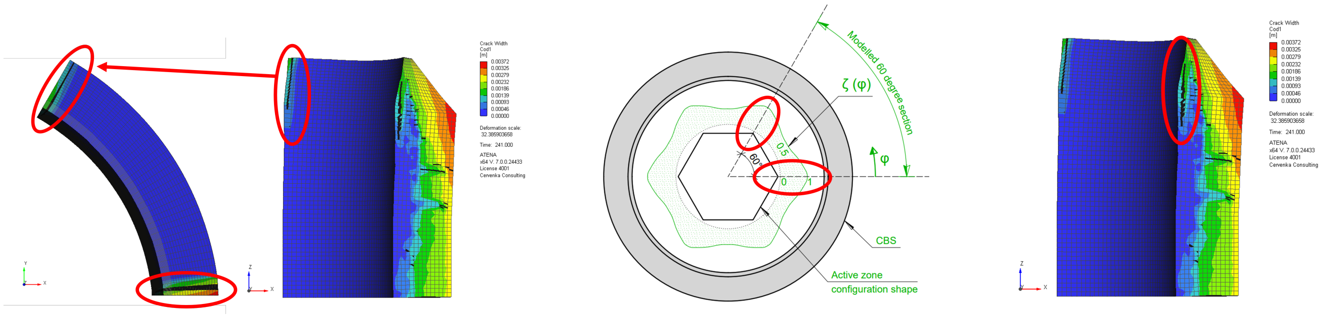
Since this particular CBS structure is non-load-bearing, two types of loads were applied: self-weight and so-called RIVE (Radiation-Induced Volumetric Expansion due to neutron fluence). The volumetric expansion was introduced as an initial strain boundary condition applied to each element. The distribution of this strain is shown in Fig. 1c. The RIVE mechanism and its implementation are described in more detail in previous work (Kovář (2023, 2024)).

The analysis focuses on crack formation and the overall damage mechanism. The following figures (Fig. 2) show the state of cracking in the concrete after 60 years of operation. As expected, dominant radial cracks appear in areas of highest radiation intensity (see Fig. 2a and Fig. 2b). Interestingly, however, the cracks appear also parallel to the inner surface of the structure (Fig. 2c), indicating a possible "delamination" of the inner part of the shielding. This result is supported by findings from the axisymmetric model (see Fig. 3), where a similar pattern was observed. At the time, it was unclear whether the effect was real or a result of mesh dependency and size effect. Whether this behavior is indeed caused by a size effect remains a subject for further investigation.

3. RBSM

Simultaneously with the continuous model development, a program based on a discrete modeling approach, specifically the Rigid-Body-Spring Model (RBSM), is also being developed. The previous work (Kovář (2024)) introduced a 2D linear RBSM formulation along with a discretization algorithm tailored for such models.

As a next step, the model is being extended to three dimensions, as 2D representations are increasingly inadequate for modern, complex structural problems. However, the transition to 3D in discrete models presents significant challenges, not only in generating the 3D Voronoi tessellation itself, but also in handling the connections between all facets, managing neighboring cell relationships, and implementing efficient indexing across the structure. These tasks introduce a level of geometric and algorithmic complexity that is significantly greater than in the



(a) *Radial cracks* (b) *Neutron fluence intensity* (c) *Cracks indicating delamination*

Figure 2: *Verification 3D model results: (a) Radial cracks, (b) neutron fluence intensity, (c) crack pattern suggesting delamination.*

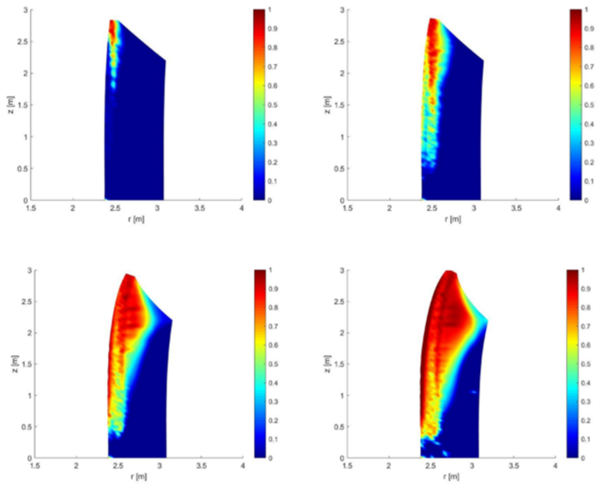


Figure 3: *Results of the axisymmetric model analysis (Kovář (2023))*

2D case. Unlike continuous models, where established meshing techniques can often be adapted with fewer complications, discrete models demand more intricate handling of geometry and interaction definitions.

Currently, the program is capable of producing satisfactory discretization for prismatic shapes (as shown in Fig. 4 and Fig. 5). However, the rounded boundary surfaces of cylindrical shapes remain problematic. The resulting meshes often contain irregularly shaped or oddly proportioned cells, which leads to unsatisfying geometries that can negatively impact both visualization and analysis (see Fig. 5). This limitation must be addressed in future development to allow modeling of standard experimental specimens, such as concrete cylinders, which are commonly used to determine the properties of the material.

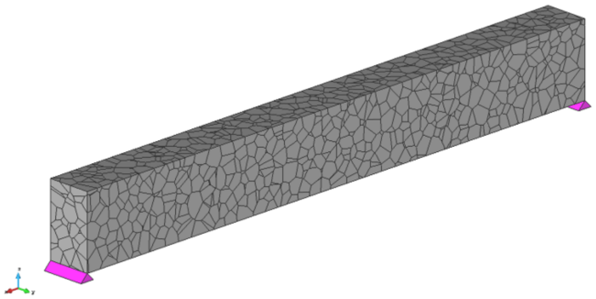


Figure 4: *Voronoi tesselation of 3D space - beam element*

To verify the basic behavior of the model, patch tests are currently being conducted at the simplest level—using two adjacent cubic cells connected by a set of springs. In these tests, one cell is fully constrained in all degrees of freedom, while the other is subjected to various force and moment loads. These experiments serve to evaluate different configurations of spring sets (corresponding constitutive laws) used to model the interaction between cells (based on Bolander et al. (2021), Asahina et al. (2015)).

Fig. 6a and Fig. 6b illustrate a comparison between two such configurations. The key difference lies in the inclusion of rotational springs. Although rotational springs are designed to directly capture rotational behavior, they are complex to implement and can introduce difficulties in subsequent stress and damage analyses. The results suggest that an alternative approach may be more suitable: using only tangential springs in combination with several normal springs placed at multiple points along the common facet between neighboring cells. This configuration effectively increases the number of integration points, offering a distributed approximation of rotational effects without explicitly requiring rotational springs (see Fig. 7, Yao et al. (2022)).

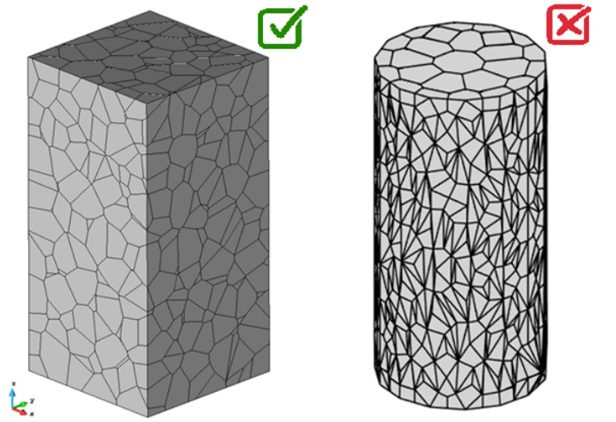
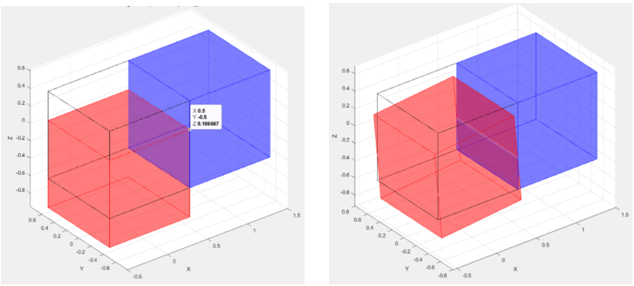


Figure 5: *Voronoi tesselation of 3D space - prismatic vs cylindrical element*



(a) *Patch test without a rotation spring*

(b) *Patch test with a rotation spring*

Figure 6: *Comparison of RBSM patch tests with and without rotational springs.*

4. SUMMARY AND CONCLUSIONS

This paper presents ongoing progress in the development of custom numerical tools for modeling concrete structures using both continuous and discrete approaches. The finite element model was further advanced with the implementation of creep behavior, initially based on Model Code 2010. However, due to its limitations, a transition toward the more versatile B3 creep model is planned, as it offers greater flexibility and accuracy for complex simulations. Validation of the custom-software through comparison with a verified commercial program (ATENA) has been initiated, with promising alignment in key response characteristics, particularly regarding crack development and signs of delamination behavior.

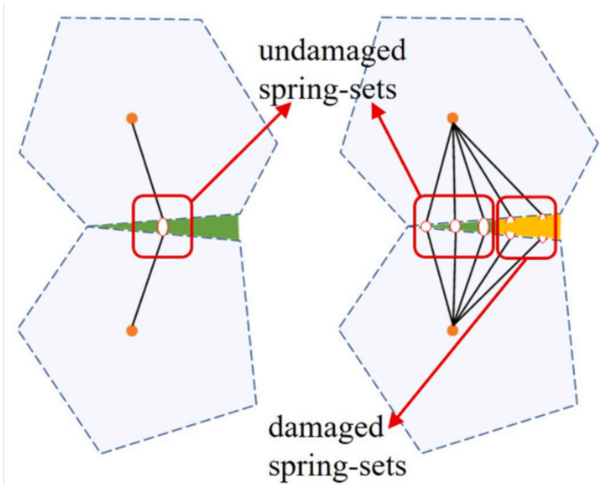


Figure 7: *Multiple normal springs ability to capture rotation*

Paralelly, development of a 3D Rigid-Body-Spring Model (RBSM) was initiated to explore discrete modeling potential. While the transition from 2D to 3D introduces substantial challenges (particularly in geometry handling and neighbor connectivity). Early patch tests suggest that a combination of tangential and multiple normal springs may offer a practical alternative to rotational springs, simplifying the formulation while preserving essential mechanical behavior.

Further development will focus on improving the 3D discretization, especially for non-prismatic geometries, and expanding the validation scope of both models. These efforts aim to build a comprehensive and flexible simulation framework suitable for advanced research and practical engineering applications involving complex concrete structures.

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