Concrete and Masonry structures 3

133CM03

Model Homework

Post-tensioned prestressed concrete bridge - assignment

Design a post-tensioned prestressed concrete bridge of a three-span arrangement. The construction is prestressed at the age of 7 days and put into operation at the age of 100 days. The durability is expected to be 100 years. The structure is loaded by the dead load $g_0 + g_{add}$ and live load q.

Individually assigned input parameters:

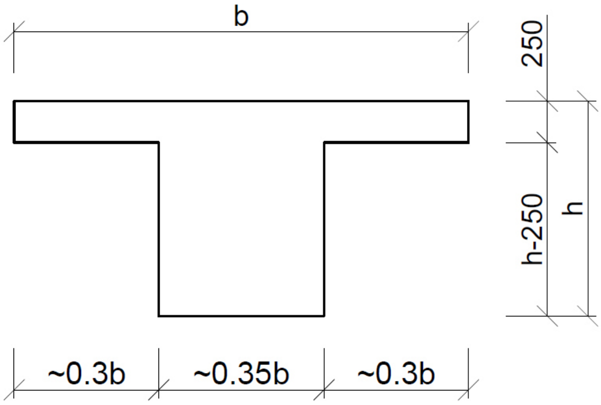
L [m]	length of the middle span
b [m]	cross-sectional width
a [-]	ratio of outer span to middle span
h [m]	cross-sectional height
$g_{add,k}$ [kN/m^2]	additional dead load except self-weight – characteristic value
q_k [kN/m^2]	live load – characteristic value

Prestressing steel: 7-wire tendons, diameter 15 mm;

$f_{pk} = 1770\ MPa$

$f_{p0.1k} = 1560\ MPa$

Cross-section shape:



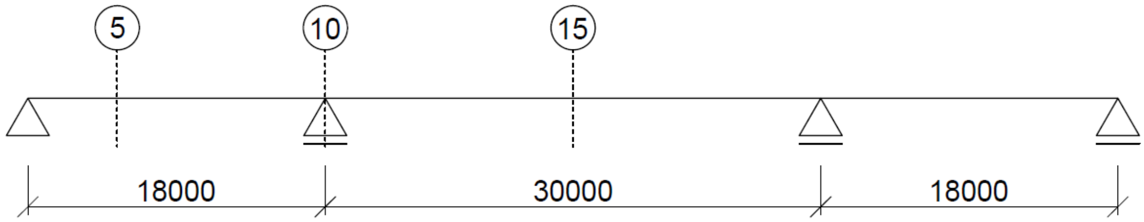
Tasks:

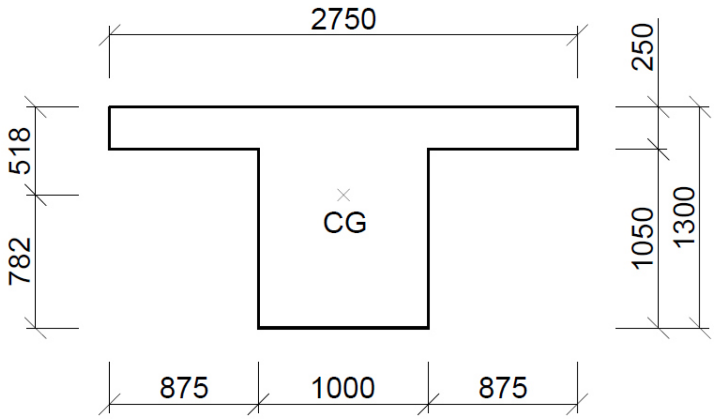
- cross-section geometry characteristics (area A; position of centre of gravity - cg; moment of inertia I)
- internal forces (N, V, M), extreme values of internal forces caused by live load
- number of tendons
- losses of prestress
- rigorous SLS assessment in decisive cross-sections (stress limit)
- ULS assessment for flexure in one of the decisive cross-sections
- ULS assessment for shear and torsion in one of the decisive cross-sections
- prestressing reinforcement drawing

Following model homework uses model input parameters. In your homework, use your own parameters obtained in the first lesson. The text in blue explains principles of the issue deeply for full understanding and/or reminds basic principles that are expected to be known from previous studies.

1. Dimensions, cross-section geometry, material characteristics

Define the geometry of the structure based on your own input parameters. The positions of the decisive cross-sections are marked as 5, 10 and 15.





Calculate following cross-sectional characteristics:

Area $A = 1.738\ m^2$

Distance between cg and bottom surface $e_b = 0.782\ m$

Distance between cg and top surface $e_t = 0.518\ m$

Moment of inertia in vertical axis $I = 0.276\ m^4$

Characteristics of a T-shaped cross-section can be calculated by dividing the T-shape into two rectangles of areas $A_1 = h_1 * b_1 = 0.25*2.75 = 0.6875\ m^2$ and $A_2 = h_2 * b_2\ 1.05*1 = 1.05\ m^2$. The distance between the upper part of the T-shape and the cg of rectangle 1 is $e_{t,1} = 250/2 = 125\ mm$, for rectangle 2 it is $e_{t,2} = 250 + 1050/2 = 775\ mm$. The centre of gravity of the T-shape is located:

$$e_t = \frac{A_1 e_{t,1} + A_2 e_{t,2}}{A_1 + A_2} = \frac{0.6875 * 0.125 + 1.05 * 0.775}{0.6875 + 1.05} = 0.518\ m$$

$$e_b = 1.05 + 0.25 - 0.518 = 0.782\ m$$

Moment of inertia of the T-shape to its centre of gravity:

$I = \frac{1}{12} b_1 h_1^3 + A_1 (e_{t,1} - e_t)^2 + \frac{1}{12} b_2 h_2^3 + A_2 (e_{t,2} - e_t)^2 = \frac{1}{12} 2.75 * 0.25^3 + 0.6875(0.125 -$ $0.5182+1121*1.053+1.050.775-0.5182=0.276\ m4$

Define the properties of used materials:

Concrete: C30/37

$f_{ck} = 30\ MPa$

$f_{cd} = f_{ck}/\gamma_M = 30/1.5 = 20\ MPa$

Prestressing steel: 7-wire tendons

diameter *15 mm*

area of one tendon $A_{p1}=150\ mm^2$

$f_{pk} = 1770\ MPa$ (ultimate stress)

$f_{p0.1k} = 1560\ MPa$ (conventional yield stress)

Assigned loading: $g_{add,k} = 2.75\ kN/m^2$

$q_k = 6\ kN/m^2$

2. Loading

The loading values assigned in the table of individual parameters are in *kN/m²*. The internal forces will be calculated on two-dimensional model, which requires the input loading to be in *kN/m* – it is necessary to multiply the assigned value by the width of the cross-section.

Linear values of loading are summarized in following tables:

Dead load	g_k [kN.m^{-1}]	γ_f [-]	g_d [kN.m^{-1}]
Self-weight g_0	43.44	1.35	58.64
Add. dead load g_{add}	2.75 x 2.75 = 7.56	1.35	10.21
Total	51.01	-	68.85

Imposed load	q_k [kN.m^{-1}]	γ_f [-]	q_d [kN.m^{-1}]
Live load q	6.5 x 2.75 = 16.5	1.5	24.75
Total	16.5	-	24.75

Loading cases listed below must be considered:

LC1 Self-weight

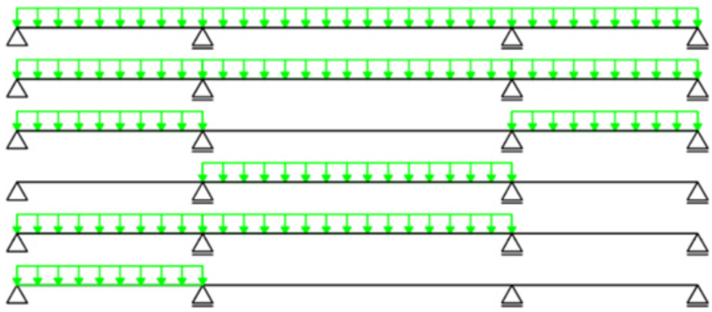
LC2 Additional dead load

LC3 Live load I

LC4 Live load II

LC5 Live load III

LC6 Live load IV



Prepare following combinations (use characteristic values: $\gamma_G = \gamma_Q = 1$):

CO1 = LC1+LC2+LC3

CO2 = LC1+LC2+LC4

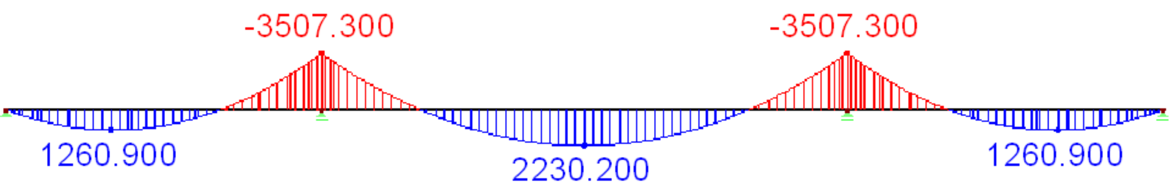
CO3 = LC1+LC2+LC5

CO4 = LC1+LC2+LC6

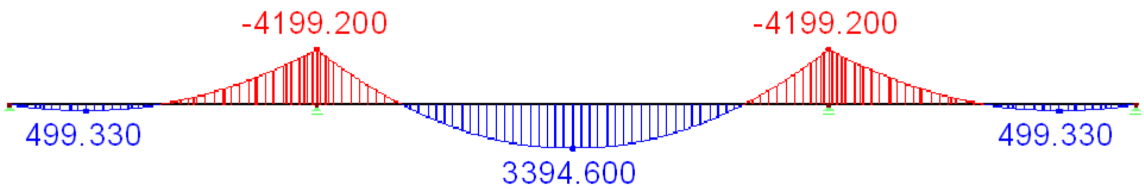
3. Internal forces

Use any software for structural analysis to calculate internal forces on the structure (N, V, M). Calculate the internal forces in combinations CO1, CO2, CO3, CO4:

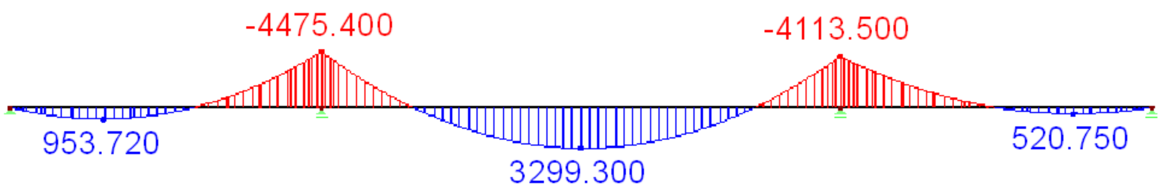
CO1:



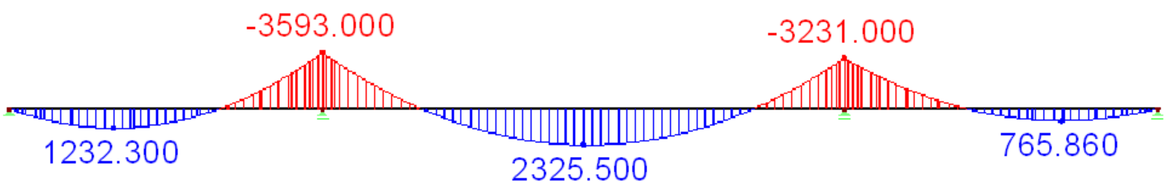
CO2:



CO3:



CO4:



(Notice: The values of internal forces for prestress design and SLS check are calculated based on characteristic values of loading. Design values will be used for ULS check only.)

3.1 Result combination

Find the result combination of maximal decisive values. Determine the position of maximal moment in outer span *x*.

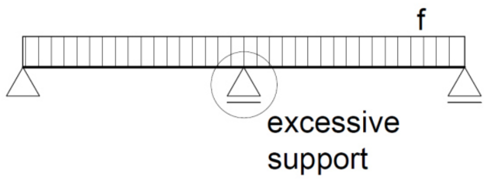
$M_{5,max} = 1260.9\ kNm;\ x = 6.113\ m$

$M_{10,min} = -4475.4\ kNm$

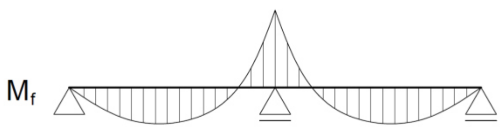
$M_{15,max} = 3394.6\ kNm$

3.2 Estimated secondary prestressing moment

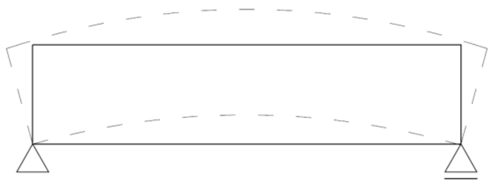
Statically indeterminate prestress structures are specific with presence of the secondary prestressing moment. Secondary prestressing moment is created by the supports that are excessive in comparison with statically determinate structures. On an example of two-span beam:



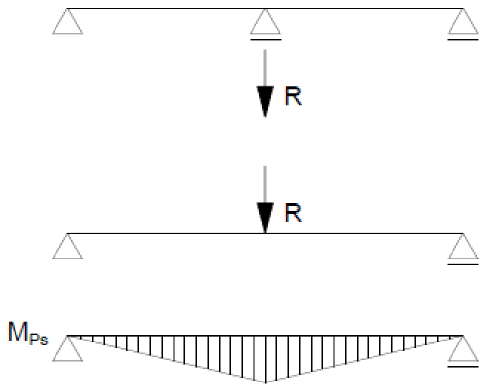
The moment caused by the loading M_f:



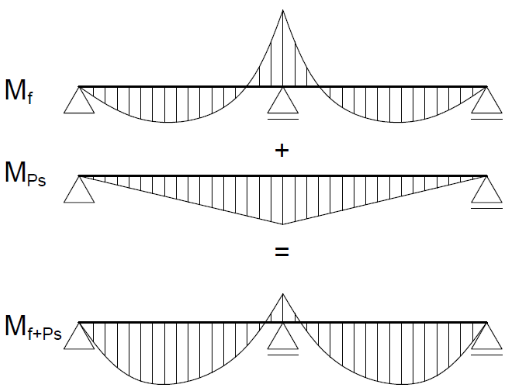
When the excessive support is neglected, the beam prestressed with parabolic reinforcement tends to deflect upwards:



However, the middle support is present and fixes the beam in its original position. This fixation causes a reaction *R* in the support. This reaction applied as a loading force to the beam creates the secondary prestressing moment M_{ps}:



The design of the prestress must consider both moment from external loading and secondary moment from prestressing $M_f + M_{ps}$:

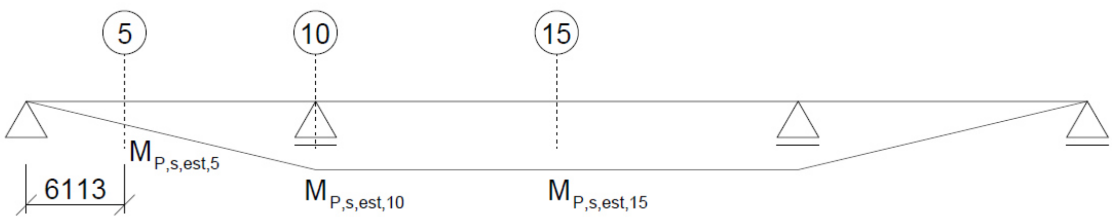


As the structure is statically indefinite, secondary prestressing moments are created (moment diagram in the picture below). Firstly, its maximal value is estimated.

$$M_{P,s,est,10} = (10 - 15)\% * M_{10,min}$$

Chosen estimation: *10 %*

$$M_{P,s,est,10} = 0.1 * M_{10,min} = 0.1 * 4475.4\ kNm = 447.5\ kNm$$

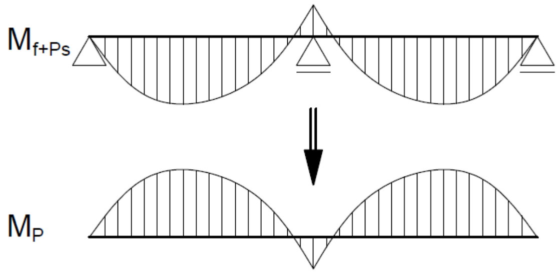


3.3 Moment values for prestress design

The position of the tendon and the value of prestressing force should be designed in a manner of balancing the effect of moment from external loading and secondary prestressing moment. Therefore, the force in the tendon should be placed on an eccentricity that creates a bending moment M_p that is as close as possible (or ideally equal) to the moments that are desired to be balanced. On an example of a two-span beam:

$$M_p \sim (M_f + M_{ps})$$

$$M_p = Pe$$



To obtain decisive moment values, both primary and secondary prestressing moment must be considered.

$$M_5 = M_{5,max} + M_{P,s,est,5} = 1260.9\ kNm + 447.5 \frac{6.113}{18} kNm = 1412.9\ kNm$$

$$M_{10} = M_{10,min} + M_{P,s,est,10} = -4475.4\ kNm + 447.5\ kNm = -4027.9\ kNm$$

$$M_{15} = M_{15,max} + M_{P,s,est,15} = 3394.6\ kNm + 447.5\ kNm = 3842.1\ kNm$$

(Notice: In cross-section 5, the value of estimated secondary prestressing moment is calculated proportionally from the value in the cross-section 10. The value of estimated secondary prestressing moment is similar in cross-section 10 and 15, as it is constant between the inner supports.)

4. Tendon design (position, prestressing force)

4.1 Eccentricity of the tendon

The tendon will be placed at the position of maximal possible eccentricity e_{10} in cross-section 10. Maximal possible eccentricity e_5 or e_{15} will be also used in cross-section 5 or 15, depending on whichever of these two has a higher value of bending moment. In this model homework, $M_{15} > M_5$, so the maximal eccentricity is used for cross-section 15.

The maximal possible eccentricity is then given by the thickness of the cover layer c of concrete. For this model homework, cover layer is given 100 mm. Do not forget to consider the diameter of the tendon duct, which is preliminarily designed to be 100 mm.

Eccentricity of the tendon (cross-section 10):

$$e_{P,10} = e_t - c - \frac{\varnothing_d}{2} = 0.518 - 0.1 - \frac{0.1}{2} = 0.368\, m$$

Eccentricity of the tendon (cross-section 15):

$$e_{P,15} = e_b - c - \frac{\varnothing_d}{2} = 0.782 - 0.1 - \frac{0.1}{2} = 0.632\, m$$

To design the eccentricity in the cross-section 5, the ratio of primary moments will be used as a proportionality coefficient (This applies only for case $M_{15} > M_5$, otherwise the process is reverse):

$$\frac{M_5}{M_{15}} = \frac{1412.9}{3842.1} = 0.37$$

$$e_5 = e_{15} * \frac{M_5}{M_{15}} = 0.632 * 0.37 = 0.234\, m$$

(The position of the tendon peak in cross-section 5 is at the place of maximal bending moment, not in the mid-span).

4.2 Prestressing force design

The aim of prestressing is to reduce the tension in concrete: THE REQUIRED STRESS AT THE TENSIONED SIDE IS ZERO (= the sum of stress created by load and prestressing is zero.) – this requirement is a base for prestress design.

$\sigma_{f,t} + \sigma_{P,t} = 0$ ... for the cross-section 10 at the top of the beam

$\sigma_{f,b} + \sigma_{P,b} = 0$ ... for the cross-section 15 at the bottom of the beam

Cross-section 10:

$$\sigma_{f,t} + \sigma_{P,t} = 0$$

where $\sigma_{f,t}$ is a stress at the top caused by loading

$\sigma_{P,t}$ is a stress at the top caused by prestressing

$$\sigma_{f,t} = \frac{M_{10}}{I_y} e_t = \frac{4027.9}{0.276} 0.518 = 7.57\, MPa\ (tension)$$

$$\sigma_{P,t} = -\frac{N_{P,10,\infty}}{A} - \frac{N_{P,10,\infty} * e_{P,10}}{I_y} \cdot e_t$$

Combining equations above, necessary prestressing force at the time of 100 years:

$$N_{P,10,\infty} = \frac{\sigma_{f,t}}{\frac{1}{A} + \frac{e_{P,10} \cdot e_t}{I_y}} \; [kN]$$

$$N_{P,10,\infty} = \frac{7\,570\,000}{\frac{1}{1.738} + \frac{0.368 * 0.518}{0.276}} = 5975 \; kN$$

Cross-section 15: (If $M_{15} < M_5$, continue with cross-section 5 instead of 15)

$$\sigma_{f,b} + \sigma_{P,b} = 0$$

where $\sigma_{f,b}$ is a stress at the bottom caused by loading

$\sigma_{P,b}$ is a stress at the bottom caused by prestressing

$$\sigma_{f,b} = \frac{M_{15}}{I_y} e_b = \frac{3842.1}{0.276} * 0.782 = 10.91 \; MPa \; (tension)$$

$$\sigma_{P,b} = -\frac{N_{P,15,\infty}}{A} - \frac{N_{P,15,\infty} * e_{P,15}}{I_y} \cdot e_b$$

Combining equations above, necessary prestressing force at the time of 100 years:

$$N_{P,15,\infty} = \frac{\sigma_{f,b}}{\frac{1}{A} + \frac{e_{P,15} * e_b}{I_y}} \; [kN]$$

$$N_{P,15,\infty} = \frac{10\,910\,000}{\frac{1}{1.738} + \frac{0.632 * 0.782}{0.276}} = 4602 \; kN$$

The decisive value of prestressing force:

There can be only one value of prestressing force in the tendon – it is necessary to choose the highest value for the design:

$$N_{P,\infty} = \max\left[N_{P,10,\infty}; N_{P,15,\infty}\right] = \max\;[5975; 4602] = 5975 \; kN$$

Due to losses of prestress, the force changes during the life of the structure. The prestressing force calculated above is designed for the age of 100 years of the construction.

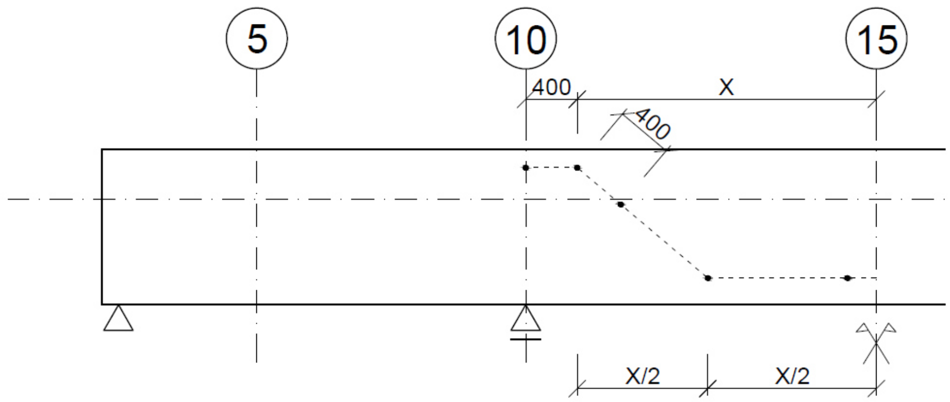
(Notice: If the extreme value of bending moment is higher in cross-section 5 than in cross-section 15, the prestressing force $N_{P,\infty}$ should be determined based on the equation $\sigma_{f,b} + \sigma_{P,b} = 0$ in cross-section 5 (not 15). Then the decisive value of prestressing force should be determined as: $N_{P,\infty} = \max\left[N_{P,10,\infty}; N_{P,15,\infty}\right]$.)

4.3 Geometry of tendon position

Create points in the distance of 400 mm from the eccentricity of cross-section 10.

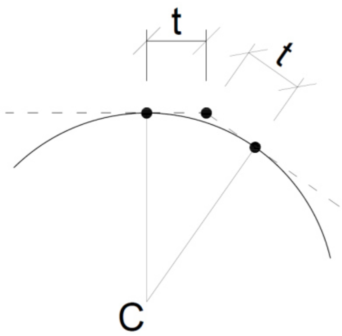
Split the interval between this point and the mid-span into halves.

Draw lines between created points.



Parabolic shape of tendon is approximated using circular arcs and straight lines. The lines drawn in the previous step are tangent to circular arcs of the tendon.

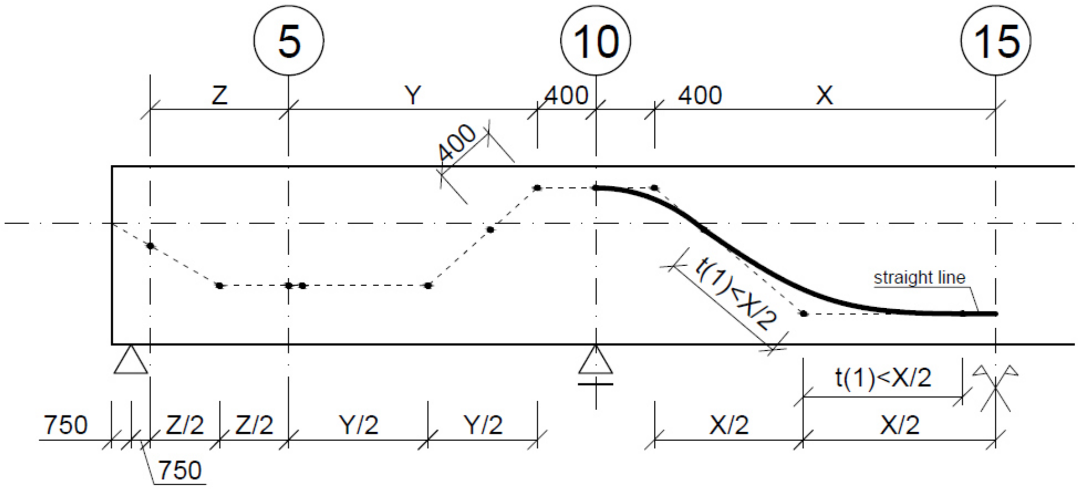
A circle can be defined by two tangents and a contact point of a circle and its tangent. When the position of one contact point is known, the second contact point on the second tangent is easily derived – it is in the same distance from the intersection of the two tangents.



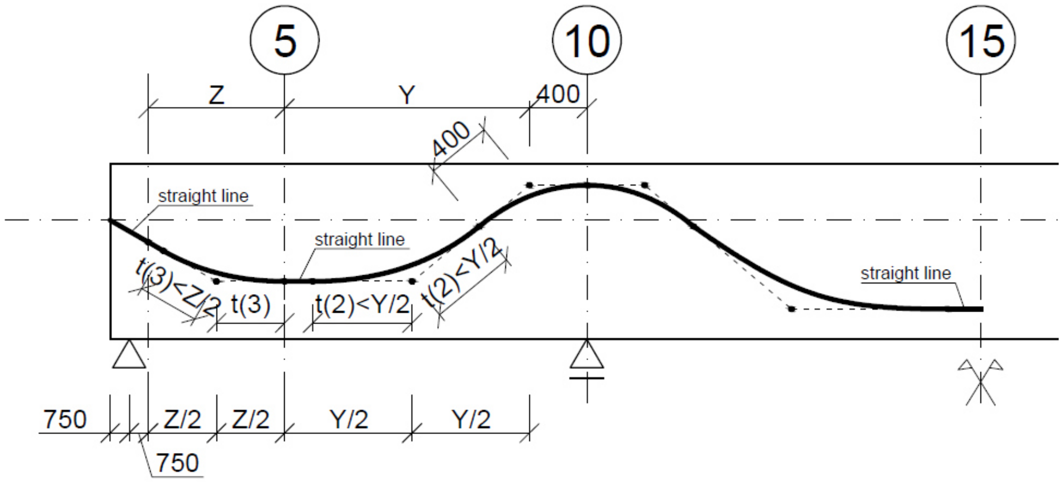
Then, the centre of the circle can be found as an intersection of two lines that are perpendicular to the tangents at the contact points.

Draw the upper (concave) circular arc from the cross-section 10 to the right side. The arc starts at the peak in the cross-section 10 and its length is up to the point placed 400 mm from the breaking point on the askew tangent.

The ending point of the right arc drawn in previous step is also the starting point of the lower (convex) arc. It is also the contact point of the lower arc with its askew tangent. Draw the lower arc using its tangents. Fill the distance between the end of the lower arch and the mid-span with a straight line.



To draw the tendon shape to the left from the cross-section 10, use analogical procedure as in previous steps. The lowest point of the lower arch must be placed in the position of maximal bending moment M_5. Notice that the first 1500 mm of the tendon must be straight. Due to the geometry of the beam, there is a straight part of the tendon before the concave arc.



5. Preliminary prestressing design

Number of tendons:

Calculate the total cross-sectional area of all tendons A_P necessary to carry the load:

$$N_{P,\infty} = \sigma_{P,\infty} A_p$$

$$A_P = \frac{N_{P,\infty}}{\sigma_{P,\infty}} \; [mm^2]$$

$$A_P = \frac{5975}{1053*10^6} = 5674 \; mm^2$$

where $\sigma_{P,\infty} = \sigma_{P,max}(1 - 25\%) = 1404 * 0{,}75 = 1053 \; MPa$

The loss of prestress is estimated to be 25% at the end of the service life of the structure (100 years of age), the precise value will be determined in further calculation.

$$\sigma_{P,max} = \min\left[0.8\, f_{pk}; 0.9\, f_{p,0,1,k}\right] =$$
$$= \min[0.8 * 1770; 0.9 * 1560] =$$
$$= \min[1416; 1404] = 1404 \; MPa$$

Calculate the number of tendons into which the total cross-sectional area will be divided:

$$n = \frac{A_P}{A_{P,1}} = \frac{5674}{150} = 37.8 \; \rightarrow \mathbf{39\ tendons}$$

Three ducts (12+12+15 tendons) will be used. (The usual amount of tendons in a duct is 12, 15, 19 – choose from these three options.)

When the design of the reinforcement is complete, cross-sectional characteristics (area, moment of inertia) should be recalculated, considering inhomogeneity of the cross-section. However, the difference between the cross-sectional characteristics of a homogeneous concrete cross-section and a cross-section with reinforcement is not very significant. Therefore it is neglected in this model homework.

6. Losses of prestress

In previous step, the losses of prestress in the end of the service life were estimated to be 25% of the initial prestress. In further text, the losses of prestress are calculated precisely.

6.1 Immediate losses of prestress

6.1.1 Loss due to friction

There is a friction between the tendon and the tendon duct in the curved parts. Also, there is friction in the straight tendon parts caused by presence of the duct supports. The friction forces along the length of the tendon are in opposite direction to the prestressing force in the tendon, decreasing it. This is the cause of the loss of prestress.

$$\Delta\sigma_{P,f} = -\sigma_{P,max} * \left(1 - e^{-\mu(\alpha + kl)}\right)$$

where $\mu \dots$ friction coefficient in curved part

$\alpha \dots$ central angle of circular arc for particular interval

$k \dots$ friction coeficient in straight part (related to 1m of length)

$l \dots$ length of particular interval

μ = 0.19 (for metal duct)

k = 0.01 m^{-1}

For each cross-section, we need to know the length of the tendon and the angular increment from the beginning of the beam. (The values are measured from the drawing – see the dimensions in the drawing of prestress). These are summarized in the table:

	Segments of the tendon (up to the axis of symetry)									
	straight part 1	arc part 2	straight part 3	arc part 4	straight part 5	arc part 6	arc part 7	straight part 8	arc part 9	straight part 10
l [m]	1.507	5.362	0.368	10.74	0	0.799	0.799	0	13.915	0.332
x [m]	1.504	5.359	0.368	10.721	0	0.798	0.796	0	13.872	0.332
α [rad]	0	0.0561	0	0.1046	0	0.1046	0.1361	0	0.1361	0
Δα [rad/m]	0	0.0105	0	0.0097	0	0.1309	0.1703	0	0.0098	0

angles: (cross-section 5) $\alpha = \sum_1^2 \alpha = 0.0561\, rad$

(cross-section 10) $\alpha = \sum_1^6 \alpha = 0.2653\, rad$

(cross-section 15) $\alpha = \sum_1^{10} \alpha = 0.5375\, rad$

lengths: (cross-section 5) $l = \sum_1^2 l = 6.869\, m$

(cross-section 10) $l = \sum_1^6 l = 18.776\, m$

(cross-section 15) $l = \sum_1^{10} l = 33.822\, m$

Loss of prestress due to friction:

$$\Delta\sigma_{P,f,5} = -1404 * \left(1 - e^{-0.19*(0.0561+0.01*6.869)}\right) = -34\, MPa$$

$$\Delta\sigma_{P,f,10} = -1404 * \left(1 - e^{-0.19*(0.2653+0.01*18.776)}\right) = -116\ MPa]$$

$$\Delta\sigma_{P,f,15} = -1404 * \left(1 - e^{-0.19*(0.5375+0.01*33.822)}\right) = -215\ MPa$$

6.1.2 Anchorage set loss

At the moment of anchoring, the anchor wedge slips into the anchor head, causing length reduction of the tendon, which leads to the loss of prestress. The slip is decreased by the friction – that means that the effect of slip decreases with the length of the tendon.

Initial slip: $w = 0.005\ m$

Hooke's law applies:

$$\Delta\sigma = E_p \Delta\varepsilon = E_p \frac{w}{L}$$

$$w = \frac{\Delta\sigma L}{E_p} = \frac{A_w}{E_p}$$

$$A_w = w * E_P = 0.005 * 195\,000 = 975\ MPa$$

where A_w is the area under the stress-length curve

changes of stress related to 1m of length in straight tendon (part 1,3 5):

$$p_{1,3,5} = \sigma_{P,0,max} * \left(1 - e^{-\mu(\Delta\alpha_{1,3,5}+kl)}\right) = 1404 * \left(1 - e^{-0.19(0+0.01*1)}\right) = 2.665\ MPa$$

changes of stress related to 1m of length in the first arc (part 2):

$$p_2 = \sigma_{P,0,max} * \left(1 - e^{-\mu(\Delta\alpha_2+kl)}\right) = 1404 * \left(1 - e^{-0.19(0.0105+0.01*1)}\right) = 5.448\ MPa$$

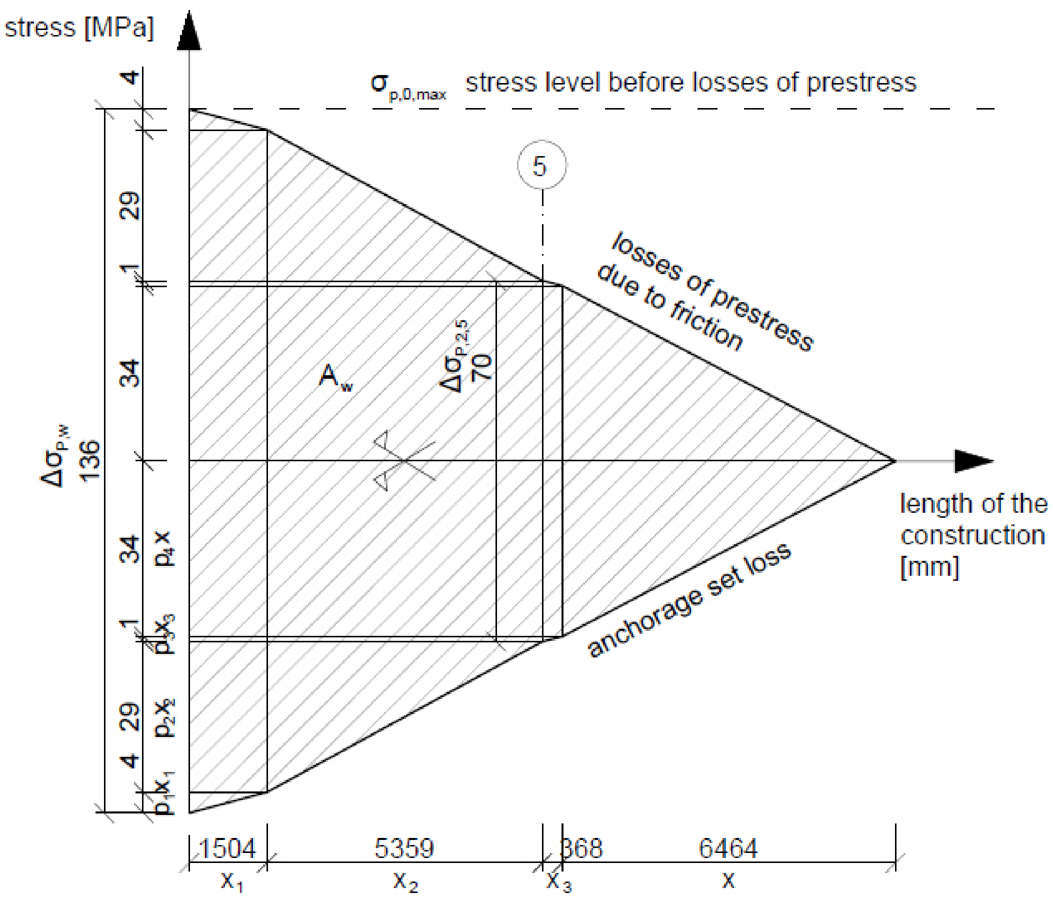
where: $\Delta\alpha_2 = \frac{\alpha_2}{L_2} = \frac{0.0561}{5.362} = 0.0105\ rad/m$

changes of stress related to 1m of length in the second arc (part 4):

$$p_4 = \sigma_{P,0,max} * \left(1 - e^{-\mu(\Delta\alpha_4+kl)}\right) = 1404 * \left(1 - e^{-0.19(0.0097+0.01*1)}\right) = 5.256\ MPa$$

where: $\Delta\alpha_4 = \frac{\alpha_4}{L_4} = \frac{0.1046}{10.74} = 0.0097\ rad/m$

The effect of anchorage set loss is assumed to be eliminated in the second arc of the tendon. The position of the elimination point is marked as x:



Area under the stress-length curve:

$$A_w = \quad 2\left(\frac{1}{2}x_1p_1 * x_1 + x_1(x_2p_2 + x_3p_3 + xp_4)\right) + \qquad \text{(straight – part 1)}$$

$$+2\left(\frac{1}{2}x_2p_2 * x_2 + x_2(x_3p_3 + xp_4)\right) + \qquad \text{(arc – part 2)}$$

$$+2\left(\frac{1}{2}p_3x_3 * x_3 + x_3(xp_4)\right) + \qquad \text{(straight – part 3)}$$

$$+2(\frac{1}{2}xp_4 * x) \qquad \text{(arc – part 4)}$$

$$5.256x^2 + 76.009x - 710.868 = 0$$

The solution of x

$$x = 6.464\ m$$

Horizontal length of the tendon where the anchorage set loss is effective x_w:

$$x_w = x_1 + x_2 + x_3 + x = 1.504 + 5.359 + 0.368 + 6.464 = 13.695\ [m]$$

The result shows that the effect of anchorage set loss will affect only the cross-section 5 – the position of x_w does not reach cross-section 10.

Total anchorage set loss:

$$\Delta\sigma_{P,w} = -2(x_1p_1 + x_2p_2 + x_3p_3 + xp_4) = -136\ MPa$$

Anchorage set loss in cross-section 5:

$$\Delta\sigma_{P,w,5} = -\left(\Delta\sigma_{P,w} - 2(x_1p_1 + x_2p_2)\right) = -(136 - 2(1.504 * 2.665 + 3.359 * 5.448 = -70\ MPa$$

Stresses in tendons after immediate losses – summary:

$$\sigma_{P,0} = \sigma_{P,0,max} + \Delta\sigma_{P,f} + \Delta\sigma_{P,w}$$

Cross-section	$\sigma_{P,0,max}$	$\Delta\sigma_{P,f}$	$\Delta\sigma_{P,w}$	$\sigma_{P,0}$	μ
5	1404	-33	-70	1301	0.735
10	1404	-116	0	1288	0.728
15	1404	-215	0	1189	0.672

(Notice: μ in this table does not stand for the friction coefficient)

$\mu = \frac{\sigma_{P,0}}{f_{pk}}$, where $f_{pk} = 1770\ MPa$

6.2 Long-term losses of prestress

6.2.1 Loss due to relaxation of prestressing reinforcement

Metal reinforcement relaxes when elongated. This means, that the stress in the tendon decreases over time even though their elongation was not reduced.

$$\Delta\sigma_{p,r} = -\sigma_{P,0} * 0.66 * 2.5 * e^{9.1\mu} \left(\frac{t}{1000}\right)^{0.75(1-\mu)} * 10^{-5}$$

where t is the time after prestressing *[hours]*

Loss of prestress at the time of 100 days:

Cross-section 5

$$\Delta\sigma_{P,r.5,100} = -1301 * 0.66 * 2.5 * e^{9.1*0.735} \left(\frac{2400}{1000}\right)^{0.75(1-0.735)} * 10^{-5} = -21\ MPa$$

Cross-section 10

$$\Delta\sigma_{P,r,10,100} = -1288 * 0.66 * 2.5 * e^{9.1*0.728} \left(\frac{2400}{1000}\right)^{0.75(1-0.728)} * 10^{-5} = -19\ MPa$$

Cross-section 15

$$\Delta\sigma_{P,r,15,100} = -1189 * 0.66 * 2.5 * e^{9.1*0.672} \left(\frac{2400}{1000}\right)^{0.75(1-0.672)} * 10^{-5} = -12\ MPa$$

Loss of prestress at the end of the service life (100 years):

Total relaxation $\Delta\sigma_{P,r,\infty}$for t = 500 000 h (*approx.* 57 *years*) – it is assumed that after this time the relaxation has no effect:

Cross-section 5

$$\Delta\sigma_{P,r.5,\infty} = -1301 * 0.66 * 2.5 * e^{9.1*0.735}\left(\frac{500000}{1000}\right)^{0.75(1-0.735)} * 10^{-5} = -59\ MPa$$

Cross-section 10

$$\Delta\sigma_{P,r,10,\infty} = -1288 * 0.66 * 2.5 * e^{9.1*0.728}\left(\frac{500000}{1000}\right)^{0.75(1-0.728)} * 10^{-5} = -57\ MPa$$

Cross-section 15

$$\Delta\sigma_{P,r,15,\infty} = -1189 * 0.66 * 2.5 * e^{9.1*0.672}\left(\frac{500000}{1000}\right)^{0.75(1-0.672)} * 10^{-5} = -45\ MPa$$

6.2.2 Loss due to creep

Concrete as a material is slightly viscous. As a result, its deformation increases even though the loading conditions are constant. As the concrete creeps, there is loss of prestress in the prestressing reinforcement.

Force in the tendon after prestressing:

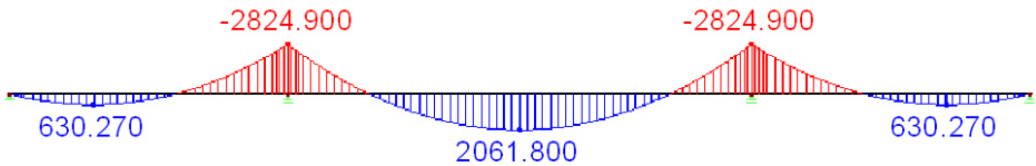
$$N_{P,0,5} = \sigma_{P,0,5} * n * A_{P1} = 1301 \cdot 39 \cdot 150 = 7612\ kN$$

$$N_{P,0,10} = \sigma_{P,0,10} * n * A_{P1} = 1288 \cdot 39 \cdot 150 = 7536\ kN$$

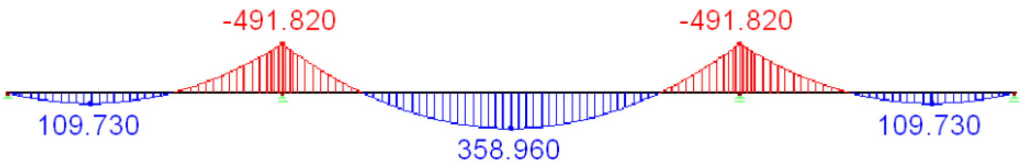
$$N_{P,0,15} = \sigma_{P,0,15} \cdot n \cdot A_{P1} = 1189 \cdot 39 \cdot 150 = 6954\ kN$$

Bending moments caused by dead load and secondary prestressing moment:

Moment caused by g_0:



Moment caused by g_{add}:



$$M_{g+P,s,est,5} = 630.75 + 109.73 + 447.50\frac{6.113}{18} = 892\ kNm$$

$$M_{g+P,s,est,10} = -2824.90 - 491.82 + 447.50 = -2869\ kNm$$

$$M_{g+P,s,est,15} = 2061.80 + 358.96 + 447.50 = 2868\ kNm$$

Elastic stress in concrete at the place of cg of the tendon:

$$\sigma_{g+P,s,est,cP} = -\frac{N_{P,0}}{A} - \frac{N_{P,0}e_P}{I}e_P + \frac{M_{g+P,s,est}}{I}e_P$$

$$\sigma_{g+P,s,est,cP,5} = -\frac{7.612}{1.757} - \frac{7.612*0.235^2}{0.276} + \frac{0.860}{0.276}0.235 = -5.14\ MPa$$

$$\sigma_{g+P,s,est,cP,10} = -\frac{7.536}{1.757} - \frac{7.536*0.368^2}{0.276} + \frac{2.823}{0.276}0.368 = -4.02\ MPa$$

$$\sigma_{g+P,s,est,cP,15} = -\frac{6.954}{1.757} - \frac{6.954*0.632^2}{0.276} + \frac{2.822}{0.276}0.632 = -7.51\ MPa$$

Loss of prestress due to creep:

$$\Delta\sigma_{P,c} = -\frac{E_p}{E_c(\tau)}\sigma_{cP}^{g+P} * \varphi$$

where τ… time of prestressing $\tau = 7$ day

t… time in the life of the beam $t_{100} = 100\ days$, $t_\infty = 100\ years$

The values of the Young's modulus at the time of 7 days ($E_c(\tau)$) and the creep coefitient of the concrete ($\varphi(t;\tau)$) are usually determined using software. For the purpose of this homework, the values are given defaultly:

$$E_c(7) = 21\ 700\ MPa$$

$$\varphi(100\ days) = 0.8$$

$$\varphi(100\ years) = 2.8$$

Loss of prestress due to creep at the time of 100 days:

$$\Delta\sigma_{P,c,5,100} = -\frac{195000}{21700} * 5.14 * 0.8 = -37\ MPa$$

$$\Delta\sigma_{P,c,10,100} = -\frac{195000}{21700} * 4.02 * 0.8 = -30\ MPa$$

$$\Delta\sigma_{P,c,15,100} = -\frac{195000}{21700} * 7.51 * 0.8 = -54\ MPa$$

Loss of prestress due to creep at the time of 100 years:

$$\Delta\sigma_{P,c,5,\infty} = -\frac{195000}{21700} * 5.14 * 2.8 = -129\ MPa$$

$$\Delta\sigma_{P,c,10,\infty} = -\frac{195000}{21700} * 4.02 * 2.8 = -106\ MPa$$

$$\Delta\sigma_{P,c,15,\infty} = -\frac{195000}{21700} * 7.51 * 2.8 = -189\ MPa$$

6.2.3 Loss due to shrinkage of concrete

Concrete shrinks over time, as the water leaves the matrix. As a result of shrinkage, there is a loss of prestress in the tendons.

$$\Delta\sigma_{P,s} = -E_p\left(\varepsilon_c^S(t) - \varepsilon_c^S(\tau)\right)$$

The shrinkage coefficient is usually determined using software. For the purpose of this homework, the values are given defaultly:

$$\varepsilon_c^S(\tau = 7days) = 8.22 * 10^{-6}$$

$$\varepsilon_c^S(t = 100\ days) = \ 56.33 * 10^{-6}$$

$$\varepsilon_c^S(t = 100\ years) = \ 439.51 * 10^{-6}$$

Loss of prestress due to shrinkage at the time of 100 days:

$$\Delta\sigma_{P,s,100} = -E_p\left(\varepsilon_c^S(100) - \varepsilon_c^S(7)\right) = \ -195000(56.33 * 10^{-6} - 8.22 * 10^{-6}) = -9\ MPa$$

Loss of prestress due to shrinkage at the time of 100 years:

$$\Delta\sigma_{P,s,\infty} = -E_p\left(\varepsilon_c^S(36500) - \varepsilon_c^S(7)\right) = \ -195000(439.51 * 10^{-6} - 8.22 * 10^{-6}) = -84\ MPa$$

Stresses in tendons after long-term losses – summary:

$$\sigma_{P\infty} = \sigma_{P0} + \Delta\sigma_{P,r} + \Delta\sigma_{P,c} + \Delta\sigma_{P,s}$$

Summary of stresses in cross-sections 5, 10, 15 at different times of service life:

Stresses and forces			Cross-section		
			5	10	15
Input stress $\sigma_{P,0,max}$ [MPa]			1404	1404	1404
Immediate losses		Friction $\Delta\sigma_{P.f}$ [MPa]	-33	-116	-215
		Anchorage set $\Delta\sigma_{P.w}$ [MPa]	-70	0.00	0.00
Stress after immediate losses $\sigma_{P,0}$ [MPa]			1301	1288	1189
Force after immediate losses $N_{P,0}$ [kN]			7612	7536	6954
Long-term losses	100 days	Relaxation $\Delta\sigma_{P.r}$ [MPa]	-21	-19	-12
		Creep $\Delta\sigma_{P.c}$ [MPa]	-37	-30	-54
		Shrinkage $\Delta\sigma_{P.s}$ [MPa]	-9	-9	-9
		Stress at the time of 100 days $\sigma_{P.100}$ [MPa]	1234	1229	1113
		Force at the time of 100 days $N_{P.100}$ [kN]	7221	7191	6513
	100 years	Relaxation $\Delta\sigma_{P.r}$ [MPa]	-59	-57	-45
		Creep $\Delta\sigma_{P.c}$ [MPa]	-129	-106	-189
		Shrinkage $\Delta\sigma_{P.s}$ [MPa]	-84	-84	-84
Stress at the end of service life $\sigma_{P,\infty}$ [MPa]			1028	1041	871
Force at the end of service life $N_{P,\infty}$ [kN]			6016	6089	5095

Percentage of stress decrease:

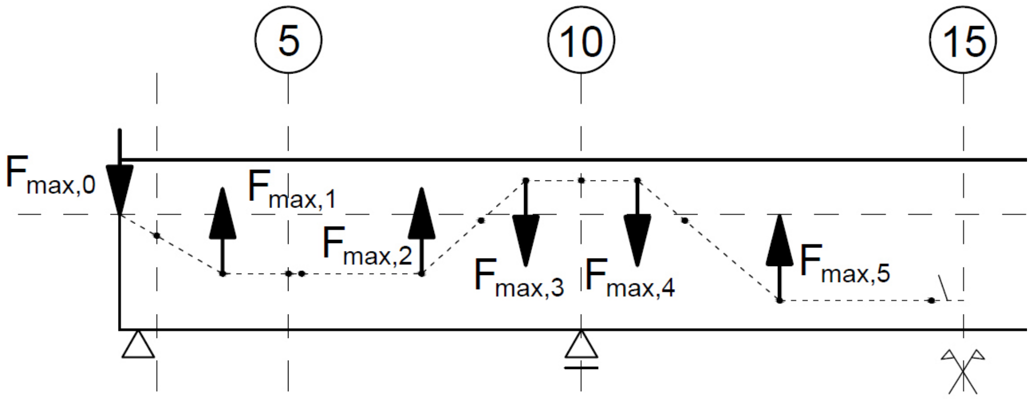
Stress	Cross-section 5	Cross-section 10	Cross-section 15
Input $\sigma_{P,0,max}$	1404 MPa = 100 %	1404 MPa = 100 %	1404 MPa = 100%
After immediate losses $\sigma_{P,0}$	1301 MPa = 93%	1288 MPa = 88%	1189 MPa = 85%
At the time of 100 days $\sigma_{P.100}$	1234 MPa = 88%	1229 MPa = 88%	1113 MPa = 79%
At the end of service life $\sigma_{P,\infty}$	1028 MPa = 73%	1041 MPa = 74%	871 MPa = 62%

7. Equivalent loads

When the exact position of the tendon duct and prestressing forces are determined, the total prestressing moment (primary + secondary) can be calculated, using the method of equivalent loads.

The principle of the method of equivalent loads is to express the effect of the prestressing on the structure as an external load. A vertical equivalent loading is created by prestressing only in the places where the tendon changes its direction (curved parts, anchors, breaking points).

The curved part (circle arc) of the tendon is defined by its tangents. When quantifying equivalent load in the curve, firstly, the vertical force F in the intersection of the tangents must be determined.



The vertical forces caused by prestressing at the points of tangent intersection and corresponding equivalent loading will be determined for the state without losses of prestress (input state, where the forces and stresses are at their maximum). That means the prestressing force in the tendon is:

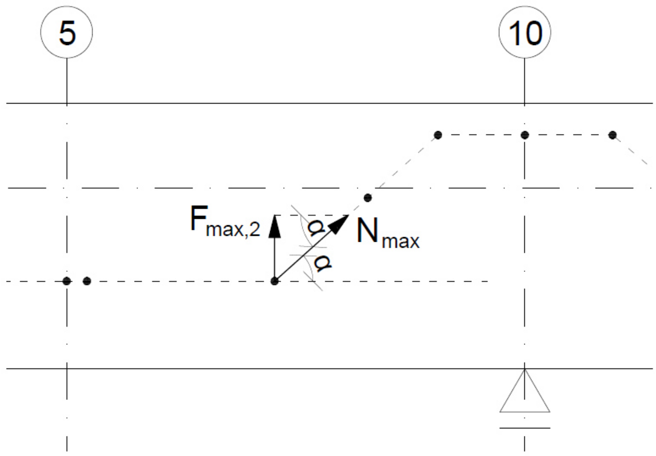
$$N_{max} = \sigma * n * A_{p,1} = 1404 * 39 * 150 = 8213\ kN$$

(The losses of prestress will be considered in the evaluation later.)

Knowing the input force in the tendon N_{max} (before losses of pretress) and the geometry:

$$F_{max} = N_{max} * \sin\alpha$$

On the example of $F_{max,2}$:



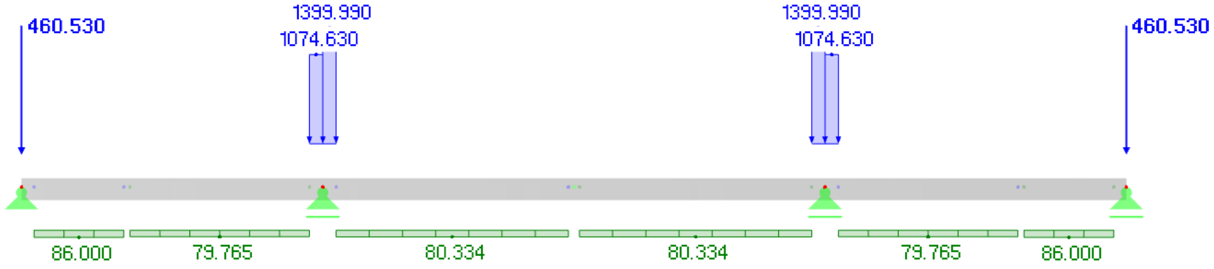
Vertical forces at the points of tangent intersection:

	α [rad]	F_{max} [kN]
tangent intersection 0 (tendon beginning)	0.0561	461
tangent intersection 1	0.0561	461
tangent intersection 2	0.1046	858
tangent intersection 3	0.1046	858
tangent intersection 4	0.1361	1114
tangent intersection 5	0.1361	1114

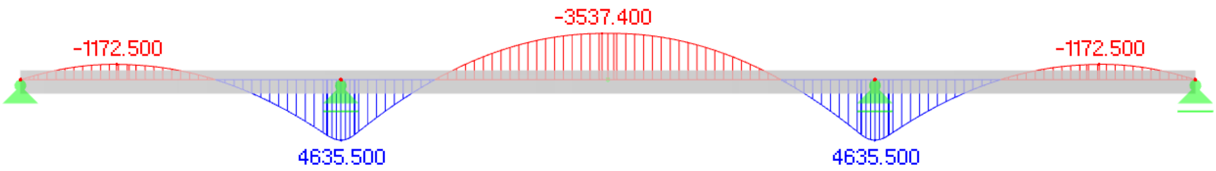
The vertical forces $F_{max,1}$ - $F_{max,5}$ are then distributed into equivalent loading $f_{max,equiv}$ over the horizontal length of each corresponding circle arc x.

	F_{max} [kN]	x [m]	$f_{max,equiv}$ [kN/m]
tangent intersection 1	461	5.359	86
tangent intersection 2	858	10.721	80
tangent intersection 3	858	0.798	1075
tangent intersection 4	1114	0.796	1400
tangent intersection 5	1114	13.872	80

The equivalent load is than applied on the model:



The bending moment caused by the equivalent load is the bending moment of prestressing $M_{p,max}$ (before losses of prestress):



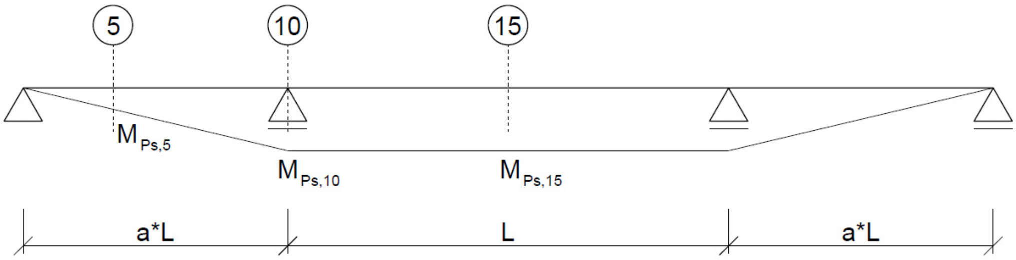
(Notice: The bending moment M_p includes both primary bending moment M_{pp} and secondary bending moment M_{ps}.)

The reactions to equivalent loading:

The vertical reactions should be balanced which means they should be of the same value. The inaccuracies in this case are caused by rounding of the subresults during the calculation. For further use, the average value of reactions will be used:

$$R = \frac{71.552 + 69.470}{2} = 70.511\ kN$$

The secondary prestressing moment $M_{ps,max}$ for the state before losses of prestress:



$$M_{ps,10,,max} = R * a * L = 70.511 * 0.6 * 30 = 1269$$

To find the values of prestressing moment M_p and secondary prestressing moment M_{ps} that include the losses of prestress, the values of bending moments obtained above will be multiplied by the percentage of stress decrease:

	Cross-section 5	Cross-section 10	Cross-section 15
$M_{p,max}$	1173 MPa	4636 MPa	3537 MPa
$M_{p,0}$	1173*0.93 = 1087 MPa	4636*0.92 = 4254 MPa	3537*0.85 = 2995 MPa
$M_{p,100}$	1173*0.88 = 1031 MPa	4636*0.88 = 4059 MPa	3537*0.79 = 2805 MPa
$M_{p,\infty}$	1173*0.73 = 859 MPa	4636*0.74 = 3437 MPa	3537*0.62 = 2194 MPa

8. SLS Check – Limitation of stress

Service Limit State check is based on stress evaluation. Stress values in decisive points of the structure must be determined and compared to limit values. To determine total stress value, the contribution of loading and prestressing must be summarized.

Elastic stress caused by loading:

at the top of the cross-section: $\sigma_{f,t} = \frac{M_f}{I} e_t$

at the bottom of the cross-section: $\sigma_{f,b} = \frac{M_f}{I} e_b$

Elastic stress caused by normal force of prestress:

at the top and bottom of the cross-section: $\sigma_{PN} = \frac{N_p}{A}$

Elastic stress caused by moment of prestress:

at the top of the cross-section: $\sigma_{PM,t} = \frac{M_P}{I} e_t$

at the bottom of the cross-section: $\sigma_{PM,b} = \frac{M_P}{I} e_b$

The principle of stress superimposing is similar for all cross-sections, all time points and for top and bottom edge of the cross-section. When superimposing stresses, be cautious about the positivity/negativity of the stress value – define correctly tensioned and compressed edge of the cross-section. Make sure you use plus sign for tension and minus sign for compression.

Stress distribution for cross-sections 5 and 15:

$$\sigma_t = -\frac{M_f}{I} e_t - \frac{N_p}{A} + \frac{M_P}{I} e_t$$

$$\sigma_b = +\frac{M_f}{I} e_b - \frac{N_p}{A} - \frac{M_P}{I} e_b$$

Stress distribution for cross-section 10:

$$\sigma_t = +\frac{M_f}{I} e_t - \frac{N_p}{A} - \frac{M_P}{I} e_t$$

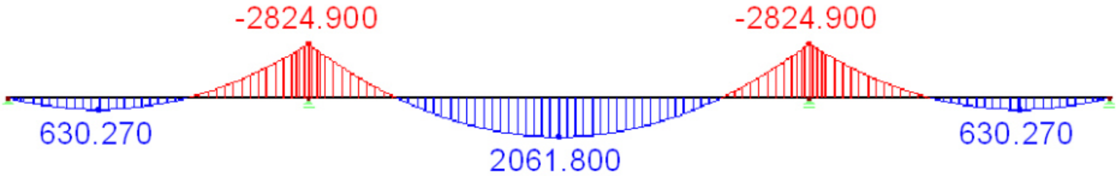
$$\sigma_b = -\frac{M_f}{I} e_b - \frac{N_p}{A} + \frac{M_P}{I} e_b$$

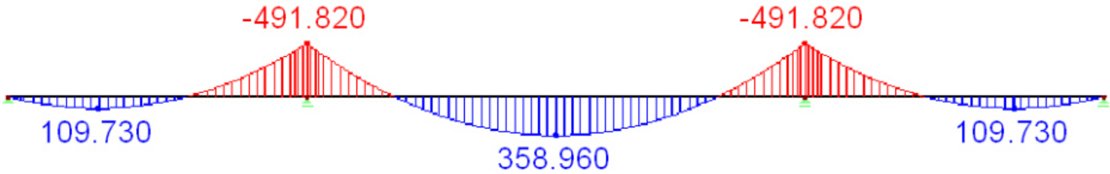
Stress values must be evaluated for:

- cross-sections 5, 10, 15
- top and bottom edge of the cross-section
- time point 0, 100, ∞

The bending moment caused by external loading M_f differs for each time point:

- at the time point 0 there is only dead load applied on the structure $g_0 + g_{add}$:





M_f [kNm]	$M_f(g_0)$	$M_f(g_{add})$	$M_f(g_{0+add})$
cross-section 5	630	110	740
cross-section 10	2825	492	3317
cross-section 15	2062	359	2421

- at the time point 100 and ∞ there is dead load and live load applied on the structure – use the extreme values of bending moment from the combinations CO1 – CO4

Stress values in the top and bottom of each cross-section in each time are summarized:

σ		time point 0	time point 100	time point ∞
5	top	-3.73	-4.59	-4.22
	bottom	-5.37	-3.50	-2.32
10	top	-6.10	-3.36	-1.55
	bottom	-1.68	-5.32	-6.45
15	top	-2.92	-4.86	-5.19
	bottom	-5.63	-2.08	**0.47**

Limiting conditions for stress values:

$\sigma \leq 0$ (stress in any part of the cross-section (top or bottom) must be negative to ensure that there is no tension (cross-section fully compressed))

$|\sigma| \leq 0.6 f_{ck} = 0.6 * 30 = 18\ MPa$ for time points 100 and ∞

$|\sigma| \leq 0.45 f_{ck} = 0.45 * 30 = 13.5\ MPa$ for time point 0

(stress in any part of the cross-section (top or bottom) must be negative to ensure that there is no tension (cross-section fully compressed))

Check all values of stress for these two conditions. The stress values that do not meet the conditions are marked red in the table. In real case, due to red values, the construction should be redesigned, however, it is not in this homework.

9. ULS Check – Flexure

In this model homework, due to the amount of work, the ULS (ultimate limit state) check for flexure is done only for the cross-section 10 at the time of 100 years. In real life structures, the ULS check should of course be done in all cross-sections at all times.

For ULS check, design values of internal forces must be used – use combination factors 1.35 for dead load and 1.5 for life load:

	characteristic moment [kNm]	design moment [kNm]
$M_f(g_0)$	-2825	-2825 * 1.35 = -3814
$M_f(g_{add})$	-492	-492 * 1.35 = -664
$M_f(q)$	-1158	-1158 * 1.5 = -1738
M_f(total)	-4475	-6216

The force in the prestressing reinforcement – use combination factors 1.0 for prestressing:

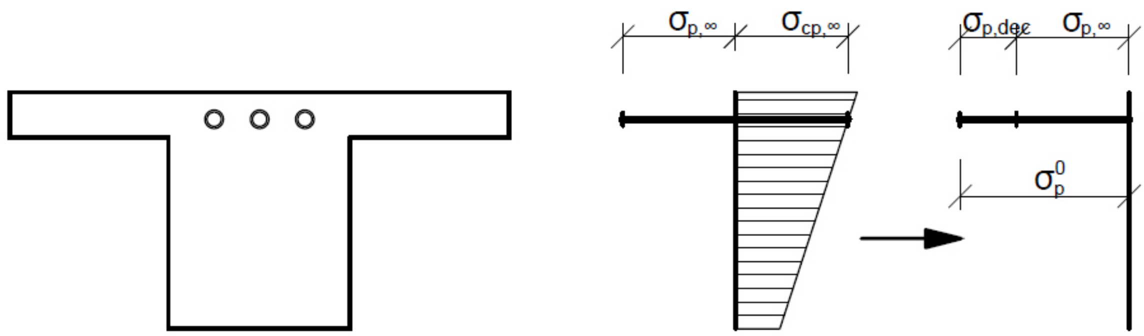
$$N_{p,\infty,d,10} = \gamma_P * N_{p,\infty,k,10} = 1.0 * 6089 = 6089\ kN$$

The secondary prestressing moment – determined using equivalent load method:

$$M_{ps,\infty,d,10} = \gamma_P * M_{ps,\infty,k,10} = 1.0 * 0.74 * 1269 = 941\ kN$$

(Notice: The secondary prestressing moment is obtained from the value of the state before losses of prestress - 1269 MPa - multiplied by the percentage of stress decrease due to losses of prestress – 74%.)

The ULS check is determined using balancing of forces on the cross-section in a state of decompression. "State of decompression" means that the compression in concrete is eliminated as the tension in the reinforcement increases:



If the compression in concrete should be eliminated from to zero, the tension in the reinforcement must be proportionally increased from $\sigma_{p,\infty}$ to $\sigma_p{}^0$ by $\sigma_{p,dec}$.

$$\sigma_{p,\infty} + \sigma_{p,dec} = \sigma_p^0$$

The stress in reinforcement after decompression σ_p^0 is called “initial stress state”. (Do not be confused as it is called “initial” even though it is after decompression. It is an initial value for further calculation.)

The stress increment in the reinforcement caused by decompression $\sigma_{p,dec}$ is derived from the stress value in concrete near the reinforcement $\sigma_{cp,\infty}$:

$$\sigma_{cp,\infty} = +\frac{M_{f,d,10}(g)}{I}e_{10} - \frac{N_{p,\infty,d,10}}{A} - \frac{N_{p,\infty,d,10}e_{10}}{I}e_{10} - \frac{M_{ps,\infty,10}}{I}e_{10} = \frac{3814+664}{0.276} * 0.368 - \frac{6089}{1.738} - \frac{6089*0.368}{0.276} * 0.368 - \frac{941}{0.276} * 0.368 = -1.77\ MPa$$

(Notice: There is a contribution of external loading moment, prestressing force, primary prestressing moment and secondary prestressing moment. The moment $M_{f10}(g)$ is caused only by dead load - live load is not included as it would cause unsafety of the ULS check.)

Then the stress increment caused by decompression:

$$\sigma_{p,dec} = \frac{E_p}{E_{cm}}\left|\sigma_{cp,\infty}\right| = \frac{195}{32} * 1.77 = 11\ MPa$$

Initial stress and corresponding strain:

$$\sigma_p^0 = \sigma_{p,\infty} + \sigma_{p,dec} = 1041 + 11 = 1052\ MPa$$

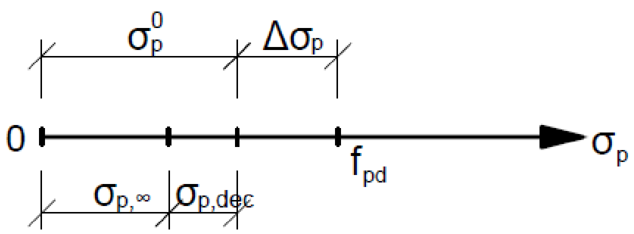
$$\varepsilon_p^0 = \frac{\sigma_p^0}{E_p} = \frac{1052}{195000} = 0.0054$$

Initial prestressing force and initial primary prestressing moment in the reinforcement (corresponding to initial stress):

$$N_p^0 = \sigma_p^0 * n * A_{p1} = 1052 * 39 * 150 = 6152\ kN$$

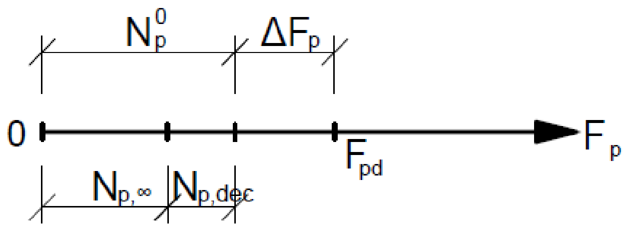
$$M_{pp}^0 = N_p^0 * e_{10} = 6152 * 0.368 = 2263\ kNm$$

The following scheme explains the stresses in the reinforcement for different states:

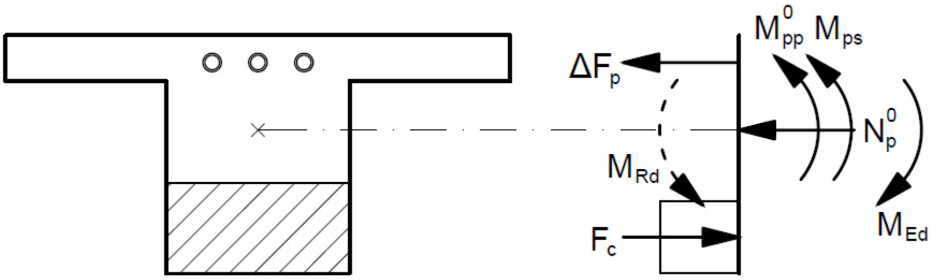


The reserve stress $\Delta\sigma_p$ is variable – when it is zero, the total stress is equal to σ_p^0; when it is at its maximum, total stress is equal to f_{pd}.

Accordingly to the stresses, the axial forces in the reinforcement:



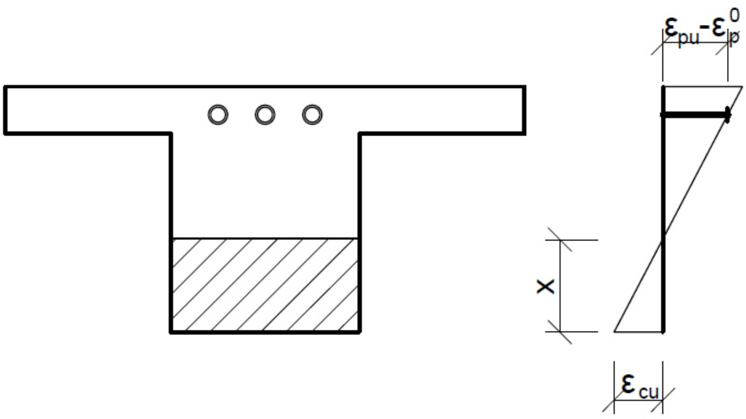
All stresses up to σ_p^0 will be considered as external force acting on the cross-section. Only stresses exceeding σ_p^0 (which means $\Delta\sigma_p$) will be considered as internal. Respectively, the same applies for forces. Based on that, the forces acting on the cross-section internally are: compressive force in concrete F_c and tensile reserve force in reinforcement ΔF_p. External forces are: initial prestressing force N_p^0, initial primary prestressing moment M_{pp}^0, secondary prestressing moment M_{ps}.



The strain distribution over the cross-section caused by internal forces at the ULS must be determined. (The key parameters to find are: the increment of strain of the reinforcement after decompression (ε_p - ε_p^0), strain of concrete ε_c. the height of compressed part of the cross-section x.) At the ULS, either the reinforcement or concrete reaches its strain limit (exceeding this limit the structure fails). Whether the reinforcement or concrete reaches the strain limit first is unknown yet. To find out, at first it is initially estimated that the both strains of reinforcement and concrete are set to their limit:

$$\varepsilon_c = \varepsilon_{cu} = 0.0035$$

$$\varepsilon_p = \varepsilon_{pu} = 0.02$$



From the geometry, the distance x for the case of first estimation is derived. Remember that the strains are evaluated only for the internal forces and, as it was stated before, all stresses (and of course the corresponding strains) up to $\sigma_p{}^0$ will be considered as external. Therefore, the strain of the reinforcement is not ε_{pu}, it is $(\varepsilon_p - \varepsilon_p{}^0)$.

$$\frac{x}{\varepsilon_{cu}} = \frac{e_{10} + e_b - x}{\varepsilon_{pu} - \varepsilon_p^0}$$

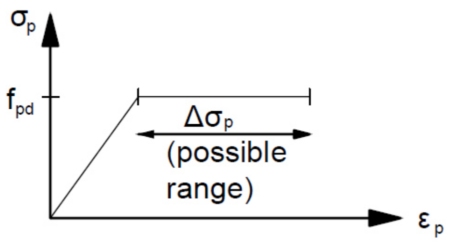
$$x = \frac{\varepsilon_{cu}(e_{10}+e_b)}{\varepsilon_{pu}-\varepsilon_p^0+\varepsilon_{cu}} = 222\ mm$$

It is presumed that the stress in the reinforcement is plastic for all stages of ULS. Therefore, it can be stated that the stress in the reinforcement is at its limit value:

$$\sigma_p = f_{pd}$$

$$\Delta\sigma_p = f_{pd} - \sigma_p^0$$

(Notice: In case the stress at any stage of ULS decreases under the limit of proportionality, the method used in this model homework is not valid, as the stresses cannot be presumed to be constant.)



Then the reserve force in the reinforcement ΔF_p is:

$$\Delta F_p = \Delta\sigma_p * n * A_{p1} = \left(f_{pd} - \sigma_p^0\right) * n * A_{p1} = (1356 - 1052) * 39 * 150 = 1784\ kN$$

where $f_{pd} = \frac{f_{p,01,k}}{\gamma_m} = \frac{1560}{1.15} = 1356\ MPa$

The axial forces on the cross-section must be balanced:

$$\Delta F_p + N_p^0 = F_c$$

where F_c is the force in concrete, that is dependent on the height of the compressed part of the cross-section x

$$F_c = 0.8 x b_w f_{cd}$$

For the x from the first estimation: $F_c = 0.8 x b_w f_{cd} = 0.8 * 0.222 * 1 * 20 = 3557\ kN$

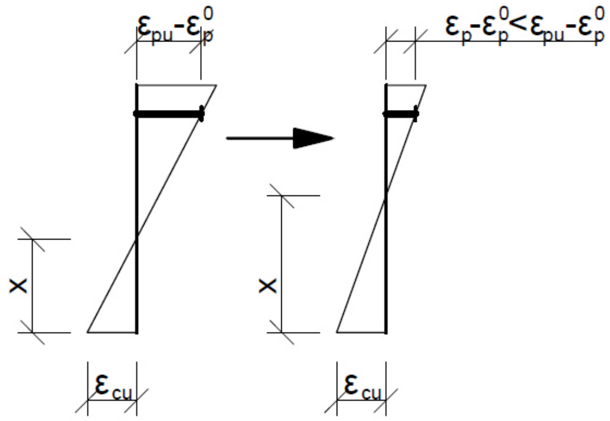
Then the balance on the cross-section:

$$\Delta F_p + N_p^0 = F_c$$

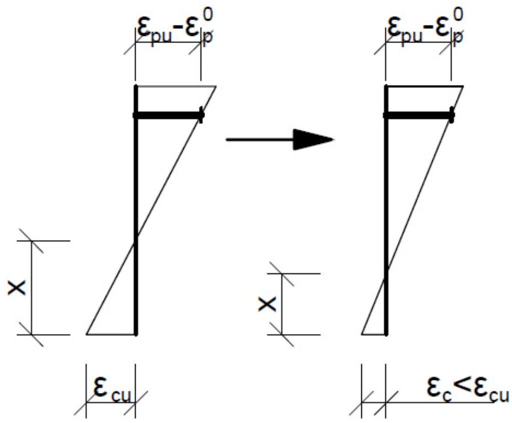
$$1784 + 6152 \neq 3557$$

As it is clear from the equation above, the force in the concrete F_c is not large enough to balance the opposed forces. That means that in fact the force F_c is larger than it was calculated in the initial estimation. As the real force F_c is larger, the real x is also larger than *223 mm* obtained from the initially estimated geometry of strain distribution.

The strains in concrete and steel were set to their limit values in the initial estimation and these values cannot be exceeded. From the geometry of the strain distribution is clear, that when the x is increased from its previous value, the strains will also change. Since the strain of concrete cannot be increased anymore, the strain of the reinforcement must be decreased. This provides us with an information that the in all steps further, the strain of concrete remains fixed value $\varepsilon_c = \varepsilon_{cu}$.



If the force F_c would have exceeded the sum of ΔF_p and N_p^0, it would have been necessary to lower it and, therefore, lower the x. In that case, the strain of the reinforcement would have remained fixed on its limit value ε_{pu} and the strain of concrete would have been lowered.



The real value of x is derived from the equilibrium of axial forces:

$$\Delta F_p + N_p^0 = F_c = 0.8 x b_w f_{cd}$$

$$x = \frac{\Delta F_p + N_p^0}{0.8 b_w f_{cd}} = \frac{1784 + 6152}{0.8 * 1 * 20} = 496\, mm$$

The previous step revealed that the strain of steel is in fact lower than the strain set in the initial estimation $\varepsilon_p < \varepsilon_{pu}$. To calculate the real value of strain of steel, use the geometry with the real x = *496 mm*:

$$\frac{x}{\varepsilon_{cu}} = \frac{e_{10} + e_b - x}{\varepsilon_p - \varepsilon_p^0}$$

$$\varepsilon_p = \frac{\varepsilon_{cu}(e_{10}+e_b-x)+x\varepsilon_p^0}{x} = \frac{\varepsilon_{cu}(e_{10}+e_b-0.496)+0.496\varepsilon_p^0}{0.496} = 0.0100$$

It is necessary to check whether this strain is larger than the strain on the limit of elasticity for steel. If not, the method used in this model homework cannot be used.

$$\varepsilon_{p,elastic} = \frac{f_{p,elastic}}{E} = \frac{f_{pd}}{E} = \frac{1356}{195000} = 0.007$$

$\varepsilon_{p,elastic} = 0.01 > \varepsilon_{p,elastic} = 0.007$ ... the method can be used

The compressive force in concrete using the real $x = 496\ mm$:

$$F_c = 0.8xb_w f_{cd} = 0.8 * 0.496 * 1 * 20 = 7936\ kN$$

Check the balance of the axial forces:

$$\Delta F_p + N_p^0 = F_c$$

$$1784 + 6152 = 7936$$

To calculate the moment of internal forces to the centre of gravity, the distances of forces F_c and ΔF_p must be determined from the geometry:

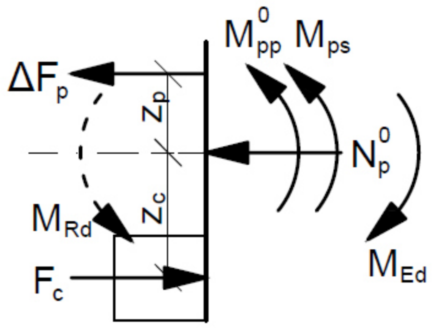
$$z_p = e_{10} = 368\ mm$$

$$z_c = e_b - 0.4x = 782 - 0.4 * 496 = 368\ mm$$

The moment resistance of internal forces:

$$M_{Rd} = F_c z_c + \Delta F_p z_p = 7936 * 0.368 + 1784 * 0.368 = 3960\ kNm$$

ULS Check:



$$M_{Rd} + M_{pp}^0 + M_{Ps} > M_{Ed}$$

$$3960 + 2263 + 941 > 6216$$

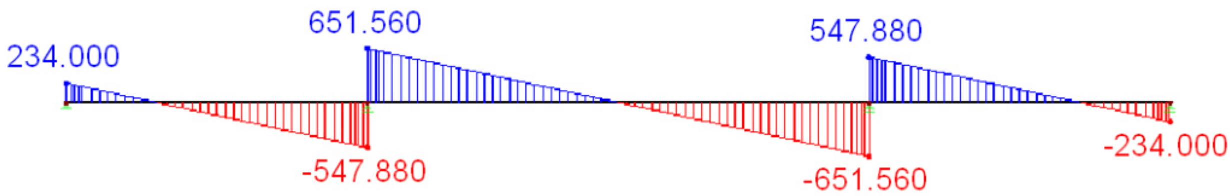
$7164\ kNm > 6216\ kNm$ ... ULS check for flexure OK

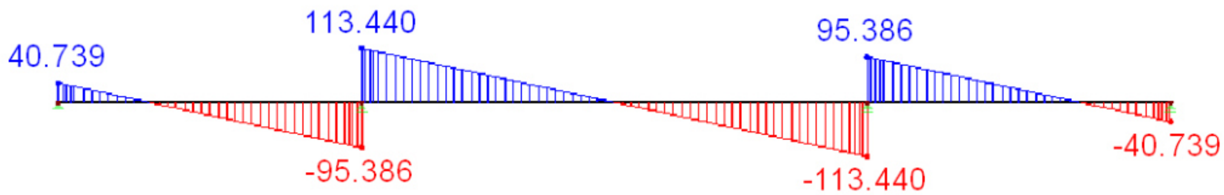
10. ULS Check – Shear and Torsion

10.1 Effect of load due to shear force

Find the extreme values of shear force for the design values of loading.

The shear forces caused by characteristic values of self-weight and additional dead load are:





Cross-section 0:

$$V_{Ek,g,0} = 234 + 41 = 275\ kN$$

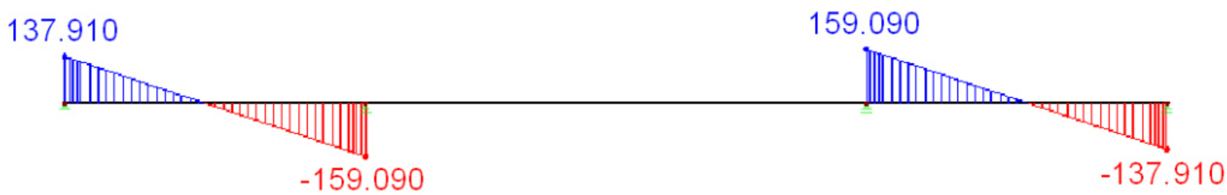
$$V_{Ed,g,0} = 275 * 1,35 = 371 kN$$

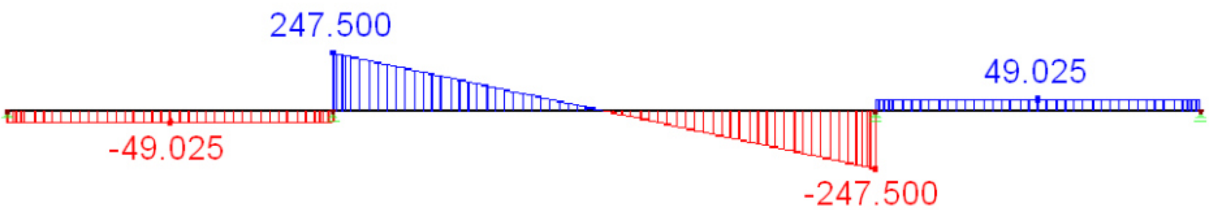
Cross-section 10:

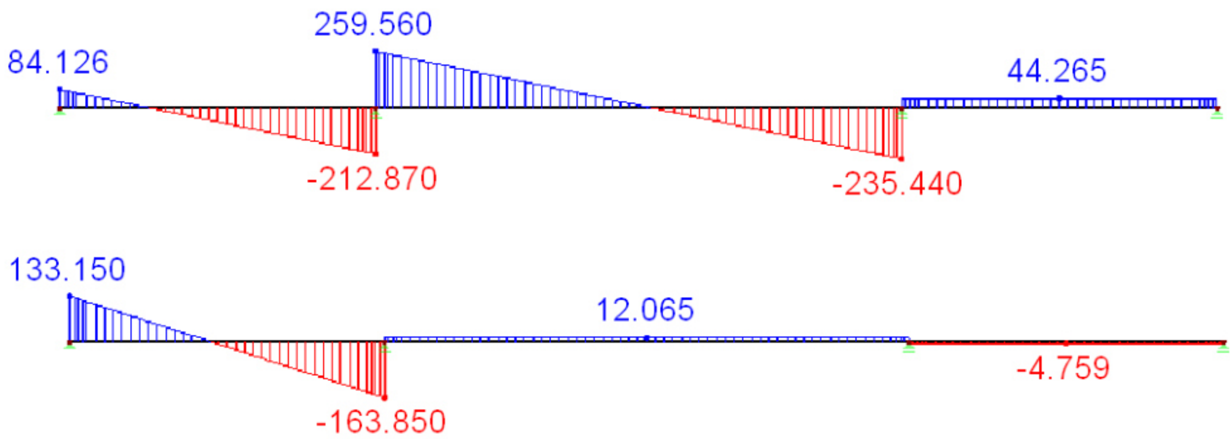
$$V_{Ek,g,10} = 652 + 113 = 765\ kN$$

$$V_{Ed,g,10} = 765 * 1,35 = 1033\ kN$$

The shear forces caused by characteristic values of live load (loading cases LC3 – LC6):







The maximal characteristic values of shear force of LC3 – LC6:

Cross-section 0:

$V_{Ek,q\ 0} = 138\ kN$

$V_{Ed,q,0} = 138 * 1,5 = 207\ kN$

Cross-section 10:

$V_{Ek,q,10} = 260\ kN$

$V_{Ed,q,10} = 260 * 1,5 = 390\ kN$

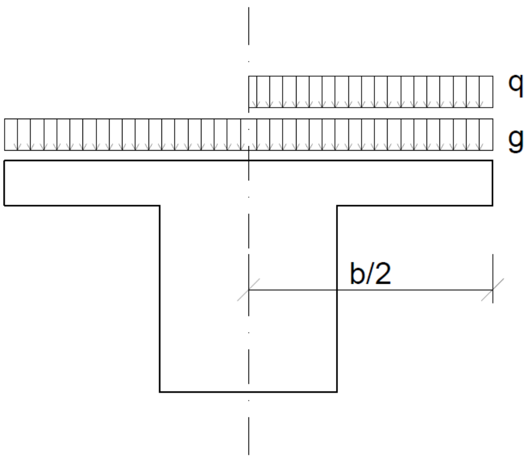
Decisive design values of shear force:

Cross-section 0: $V_{Ed,s,0} = 371 + 207 = 578\ kN$

Cross-section 10: $V_{Ed,s,10} = 1033 + 390 = 1423\ kN$

10.2 Effect of load due to torsion

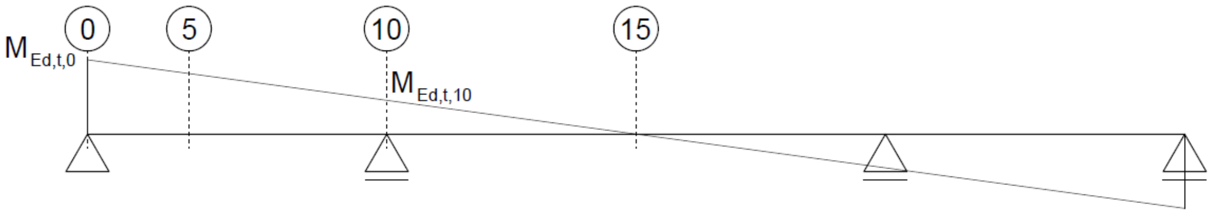
Torsion is created on a structure when the loading is applied on a cross-section asymmetrically. For that purpose, we apply the live load on only one half of the beam:



The dead load is symmetrical, which means its effect on the torsion is zero.

The torque loading is (design value): $m_{Ed,t} = \gamma_q q_k \frac{b}{2} * \frac{b}{4} = 1.5 * 6 * \frac{2.75}{8} = 3.09\ kNm/m$

We presume that the construction is fixed against torsion only by the first and last support. That means the torque along the construction length is:



$$M_{Ed,t,0} = \left(aL + \frac{1}{2}L\right) * m_{Ed,t} = 33 * 3.09 = 102\ kNm$$

$$M_{Ed,t,10} = \left(\frac{1}{2}L\right) * m_{Ed,t} = 15 * 3.09 = 46\ kNm$$

The resistance of the cross-section to the torsion is provided only by the perimeter of the cross-sectional web. Therefore, the torque must be distributed only to this part of the cross-section.

The thickness of the perimeter is:

$$t_{ef} = \frac{A}{u} = \frac{1.738}{8.1} = 0.215\ m$$

where A is cross-sectional area *(A = 1.738 m^2)*

u is perimeter of the whole cross-section *(u = 2.75 + 2*0.25 + 2*0.875 + 2*1.05 + 1 = 8.1 m)*

Make sure that $t_{ef} = 0{,}215 \geq 2c$ where c is cover layer thickness.

$$t_{ef} = 0.215 \geq 2c = 100\ \rightarrow \text{OK}$$

Torsion is related to shear and it can be expressed by it. The relationship between torque moment $M_{Ed,t}$ and shear stress τ_t it creates is:

$$\tau_t * t_{ef} = \frac{M_{Ed,t}}{2A_k}$$

where $A_k = (b - t_{ef})(h - t_{ef}) = (1 - 0.215)(1.3 - 0.215) = 0.852\ m^2$

The shear force created by torsion is:

$$V_{Ed,t} = \tau_t * t_{ef} * z = \frac{M_{Ed,t}}{2A_k}(h - t_{ef})$$

$$V_{Ed,t,0} = \frac{M_{Ed,t,0}}{2A_k}(h - t_{ef}) = \frac{46}{2 * 0.852} * (1.3 - 0.215) = 30\ kN$$

$$V_{Ed,t,10} = \frac{M_{Ed,t,10}}{2A_k}(h - t_{ef}) = \frac{102}{2 * 0.852} * (1.3 - 0.215) = 65\ kN$$

(Notice: The area where the shear stress τ_t is active is here considered without horizontal parts of the perimeter: $t_{ef} * z$. The reason for this is that only the vertical part of the force contributes to the shear force $V_{Ed,t}$.)

10.3 Shear and torsion resistance of the structure

Two conditions must be checked – condition for compressive stress in the concrete diagonal and condition for sufficient shear resistance of the shear reinforcement.

Condition for compressive stress in the diagonal:

$$V_{Rd,max} = \alpha_{cw} b_w z v_1 f_{cd} \frac{cotg\theta + cotg\alpha}{1 + cotg^2\theta}$$

where α_{cw} expresses the positive effect of prestressing

$\alpha_{cw} = 1 + \frac{\sigma_{cp}}{f_{cd}}$ for $0 < \sigma_{cp} < 0.25 f_{cd}$

$\alpha_{cw} = 1.25$ for $0.25 f_{cd} < \sigma_{cp} < 0.5 f_{cd}$

$\alpha_{cw} = 2.5\left(1 - \frac{\sigma_{cp}}{f_{cd}}\right)$ for $0.5 f_{cd} < \sigma_{cp} < 1.0 f_{cd}$

where σ_{cp} is a stress caused by prestressing force at the end of service life

$$\sigma_{cp,10} = \frac{N_{p,\infty,d,10}}{A_c} = \frac{6089}{1.738} = 3504\ kPa$$

(For simplification, use the stress value of the cross-section 10 also for the cross-section 0.)

$$f_{cd} = \frac{f_{ck}}{1.5} = \frac{30}{1.5} = 20\ MPa$$

$$0 < \sigma_{cp} = 3.504\ MPa < 0.25 f_{cd} = 0.25 * 20 = 5\ MPa$$

$$\alpha_{cw} = 1 + \frac{\sigma_{cp}}{f_{cd}} = 1 + \frac{3.504}{20} = 1.18$$

b_w is the width of the cross-section web ($b_w = 1\ m$)

z is the distance between internal forces on the cross-section, can be estimated $z = 0.9(e_b + e_{10}) = 0.9(0.782 + 0.368) = 1.035m$

v_1 is a reduction coefficient for concrete strength

$$v_1 = 0.6\left(1 - \frac{f_{ck}}{250}\right) = 0.6\left(1 - \frac{30}{250}\right) = 0.53$$

θ is the angle of compressive forces, choose in the interval of $1.25 \leq cotg\theta \leq 2.5$; $cotg\theta$ chosen 2.0

α is the angle of shear reinforcement, $\alpha = 0$

$V_{Rd,max} = 5138\ kN\ > V_{Ed,s,0} + V_{Ed.t,0} = 578 + 30 = 608\ kN \rightarrow$ OK in cross-section 0

$V_{Rd,max} = 5138\ kN\ > V_{Ed,s,10} + V_{Ed.t,10} = 1423 + 65 = 1488\ kN \rightarrow$ OK in cross-section 10

Condition for sufficient shear resistance of the shear reinforcement:

$$V_{Rd,s} = \frac{A_{sw}}{s} z f_{ywd} cotg\theta$$

kde A_{sw}/s is the area of shear reinforcement per 1m of construction length

Design a sufficient shear reinforcement:

$\frac{A_{sw}}{s_0} = \frac{314}{0.3} = 1047\ mm^2/m$ ... reinforced by *4 x Ø10mm* per *300 mm*

$\frac{A_{sw}}{s_{10}} = \frac{314}{0.15} = 2094\ mm^2/m$ ... reinforced by *4 x Ø10mm* per *150 mm*

where A_{sw} is the cross-sectional area of shear reinforcement in one row and s is distance between the rows.

The condition for minimal ratio of shear reinforcement must be met:

$$\rho_w = \frac{A_{sw}}{sb_w sin\alpha} \geq \rho_{w,min} = \frac{0.08\sqrt{f_{ck}}}{f_{ywk}} = \frac{0.08\sqrt{30}}{500} = 0.00088$$

$\rho_{w,0} = \frac{A_{sw,0}}{sb_w sin\alpha} = 0.0011\ \geq \rho_{w,min} = 0.0009\ \rightarrow$ OK in cross-section 0

$\rho_{w,10} = \frac{A_{sw,10}}{sb_w sin\alpha} = 0.0021\ \geq \rho_{w,min} = 0.0009\ \rightarrow$ OK in cross-section 10

f_{ywd} is the design value of yield stress of shear reinforcement

$$f_{ywd} = \frac{f_{ywk}}{1.15} = \frac{500}{1.15} = 435\ MPa$$

$$V_{Rd,s,0} = \frac{A_{sw,0}}{s} z f_{ywd} cotg\theta = \frac{314}{300} * 1.035 * 435 * 2 = 942\ kN$$

$$V_{Rd,s,10} = \frac{A_{sw,10}}{s} z f_{ywd} cotg\theta = \frac{314}{150} * 1.035 * 435 * 2 = 1885\ kN$$

$V_{Rd,s,0} = 942\ kN\ > V_{Ed,s,0} + V_{Ed.t,0} = 578 + 30 = 608\ kN \rightarrow$ OK in cross-section 0

$V_{Rd,s,10} = 1885\ kN\ > V_{Ed,s,10} + V_{Ed.t,10} = 1423 + 65 = 1488\ kN \rightarrow$ OK in cross-section 10